

Practice Final
MAT-16A Short Calculus

Spring 2012

Name _____

- This test is closed notes, closed book.
- Laptops and calculators are NOT allowed.
- There are 12 pages and 13 questions total.
- You can leave an answer as a numerical expression without computing the final value.
For example, this is a perfectly acceptable answer :
 $((250 - 63)/(1 - e^{(-6*3.5)})) * \ln(27/168)$. Show your work clearly !!
- The maximum score in the test is 120 points.

Signature _____

Problem	Score	Max Possible
1		12
2		13
3		5
4		15
5		10
6		10
7		10
8		10
9		8
10		7
11		7
12		8
13		5
Total		120

1. Find the following limits.

(a) **(4 pts)** (Hint : Simplify fractions first)

$$\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}.$$

(b) **(4 pts)** (Hint : Rationalize the numerator and divide out)

$$\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x^2 - 1}.$$

(c) (4 pts)

$$\lim_{x \rightarrow \infty} \frac{5x^3 - 3x}{21 + 3x^2 - 6x^3}.$$

2. Differentiate the following functions. DO NOT SIMPLIFY ANSWERS.

(a) (7 pts)

$$y = \frac{\sqrt{5 + 2x}}{\cos(3x + 1)}$$

(b) (6 pts)

$$y = (\tan(\sin(x)))^3$$

3. (5 pts) Solve the equation for all values of θ in the interval $[0, 2\pi]$:

$$\cos(\theta/2) - \cos(\theta) = 1.$$

4. Consider the function

$$f(x) = \frac{6x^2}{x - 6}.$$

(This question has three parts a), b) and c) - all three parts refer to this function.)

(a) **(5 pts)** Determine all the intervals where the function is increasing and decreasing.

(b) **(5pts)** Find the relative minimum and relative maximum of $f(x)$. You need to say at what values of x these relative extrema occur, and identify whether it is a relative minimum or relative maximum. You DO NOT need to report the value of the function at these points.

- (c) **(5pts)** Find the absolute maximum and absolute minimum values of the function in the interval $-1 \leq x \leq 2$.

5. **(10 pts)** Consider the function

$$x(x - 3)^2.$$

Determine the intervals where the function is concave upwards and concave downwards. Also indicate the points of inflection.

6. **(10 pts)** Find two numbers x, y such that $x + y = 10$ and the quantity $x^2 + 4y^2$ is minimized.

7. **(10 pts)** Find the point (x, y) on the graph of $y = 1 + \sqrt{x}$ such that its distance from $(9/2, 1)$ is minimized.

8. **(10 pts)** A rectangular box with a square base is to be constructed from two materials. The material for the square top and bottom costs \$ $3/ft^2$, and the material for the four rectangular sides costs \$ $2/ft^2$. Find the dimensions of the box with largest volume which can be made for exactly \$ 72. Clearly indicate the length, width, height and volume of the box.

9. (8 pts) Find the equation of the tangent line at $(-\frac{\pi^2}{1+\pi}, \pi)$ on the graph of

$$x \sec(y) = y^2 + xy.$$

10. (7 pts) The circumference C of a circle is increasing at the rate $\frac{dC}{dt} = 2ft/sec$. What is the rate of change $\frac{dA}{dt}$ of the area A of the circle when the radius of the circle is 6 ft? The circumference C is given by $C = 2\pi r$ and the area is given by $A = \pi r^2$ where r is the radius of the circle.

11. **(7 pts)** A point (x, y) is moving along a curve such that the rate of change of its x coordinate is given by $\frac{dx}{dt} = 3$ and the rate of change of its y coordinate is given by $\frac{dy}{dt} = -2$. Consider the line through this moving point and $(2, 5)$. Find the rate of change of the slope s of this line, when the moving point has coordinates $(1, 1)$.

12. **(8 pts)** Use the definition of the derivative

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

to find the derivative of $f(x) = \sqrt{2x-1}$. DO NOT use the rules of derivatives to find the answer. Use limits.

13. (5 pts) Consider the following function :

$$f(x) = \begin{cases} x - 2 & x < 3 \\ 5 & x = 3 \\ -x^2 + 10 & x > 3 \end{cases}$$

Does $\lim_{x \rightarrow 3} f(x)$ exist ? If the limit does exist, write the value of this limit. Is the function continuous at $x = 3$? Show all your work and explain all your answers clearly.