1. Differentiate the following functions. DO NOT SIMPLIFY ANSWERS.

(a) (4 pts)

\[
y = x^5 + \frac{4}{x^2} - 10^2
\]

\[
\frac{dy}{dx} = 5x^4 \cdot \frac{2}{x^3} = \frac{5x^4 - 8}{x^3}
\]

Both are OK!

(b) (5 pts)

\[
y = x^{-5} \sin(x)
\]

\[
\frac{dy}{dx} = -5x^{-6} \sin(x) + x^{-5} \cos(x)
\]
(c) (4 pts)

\[ y = \frac{x - 2}{3 - x^5} \]

\[ \frac{dy}{dx} = \frac{(1)(3-x^5) - (x-2)(-5x^4)}{(3-x^5)^2} \]

\[ = \frac{4x^5 - 10x^4 + 3}{(3-x^5)^2} \]

(d) (6 pts)

\[ y = (\tan(2x + 4))^3 \]

\[ \frac{dy}{dx} = 3(\tan(2x + 4))^2 (\sec^2(2x + 4))(2) \]

\[ = 6 \tan^2(2x + 4) \sec(2x + 4) \]

Both are OK.
(e) (6 pts)

\[ y = \left( \frac{x + 2}{x + 1} \right)^{-3} \]

Using the chain rule:

\[ \frac{dy}{dx} = -3 \left( \frac{x + 2}{x + 1} \right)^{-4} \left( \frac{(x+1) - (x+2)}{(x+1)^2} \right) \]

\[ = -3 \left( \frac{x + 2}{x + 1} \right)^{-4} \left( \frac{-1}{(x+1)^2} \right) \]

Everything are OK.

2. (10 pts) Solve the equation for all values of \( \theta \) in the interval \([0, 2\pi]\):

\[ \cos(2\theta) - \cos(\theta) = 0. \]

(Hint: Use a double angle formula for \( \cos(2\theta) \) to express the equation as a quadratic equation in \( \cos(\theta) \).)

\[ \cos(2\theta) = 2\cos^2\theta - 1 \]

so we get

\[ 2\cos^2\theta - 1 - \cos\theta = 0 \]

Factoring:

\[ (2\cos\theta + 1)(\cos\theta - 1) = 0 \]

so \( \cos\theta = -\frac{1}{2} \) or \( \cos\theta = 1 \)

\[ \theta = \frac{2\pi}{3}, \frac{4\pi}{3} \quad \text{or} \quad \theta = 0, 2\pi \]
3. (10 pts) Find the equation of the tangent line to the graph of the equation

\[ 2xy^3 + 5xy = 14 \]

at the point (2, 1).

Using implicit differentiation,

\[ \frac{d}{dx} \left[ 2xy^3 + 5xy \right] = \frac{d}{dx} [14]. \]

\[ \Rightarrow \left( 2x \right) \left( 3y^2 \frac{dy}{dx} \right) + (2y^3) + 5x \frac{dy}{dx} + 5y = 0 \]

\[ \Rightarrow (6xy^2 + 5x) \frac{dy}{dx} = -5y - 2y^3 \]

\[ \Rightarrow \frac{dy}{dx} = \frac{-5y - 2y^3}{6xy^2 + 5x} \]

At (2, 1) we get \( \frac{dy}{dx} = \frac{-7}{18} \).

4. (8 pts) Find the equation of the line perpendicular to the tangent line at the point with \( x = 1 \) on the graph of the function \( f(x) = x^2 - 3x \).

\[ f'(x) = 2x - 3 \]

At \( x = 1 \) \( f'(1) = 2(1) - 3 = -1 \)

Slope of the perpendicular = \( -\frac{-1}{-1} = 1 \)

Y coordinate of pt = \( 1^2 - 3(1) = -2 \).

End of perp. line: \( y - (-2) = 1(x - 1) \)

\[ y + 2 = x - 1 \]

\[ y = x - 3 \]
5. (7 pts) Find \( f''(x) \) when \( f(x) = x \sin(2x) \).

\[
f'(x) = (1) \sin(2x) + x \cos(2x)(2) \\
= \sin(2x) + 2x \cos(2x)
\]

\[
f''(x) = \cos(2x)(2) + 2 \left[ \cos(2x) + x(-\sin(2x))(2) \right] \\
= 2 \cos(2x) + 2 \cos(2x) - 4x \sin(2x)
\]

\[
= 4 \cos(2x) - 4x \sin(2x)
\]

All three are OK!

6. (10 pts) The volume \( V \) of a right circular cone is increasing at 3 \( \text{in.}^3/\text{sec} \). When the radius \( r = 1 \) inch and height \( h = 3 \) inches, the rate of change of the radius is \( \frac{dr}{dt} = 2 \) inches/sec. What is the rate of change of the height \( \frac{dh}{dt} \) at this time? The relation between the volume \( V \), radius \( r \) and height \( h \) is given by \( V = \frac{1}{3} \pi r^2 h \).

\[
\text{Related Rates:} \quad \frac{d}{dt} \left[ V \right] = \frac{d}{dt} \left[ \frac{1}{3} \pi r^2 h \right] \\
\Rightarrow \quad \frac{dV}{dt} = \frac{1}{3} \pi \left[ 2r \cdot \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt} \right]
\]

\( r = 1, \quad h = 3, \quad \frac{dV}{dt} = 3, \quad \frac{dr}{dt} = 2 \).

\[
3 = \frac{1}{3} \pi \left( 2(1)(2)(3) + (1)^2 \frac{dh}{dt} \right)
\]

\[
\Rightarrow \quad \frac{dh}{dt} = \frac{9}{\pi} - 12
\]
7. (10 pts) Suppose that a point is moving in the plane such that when the point has coordinates $(1, 3)$ the rate of change of its $x$-coordinate is $\frac{dx}{dt} = 2$ and the rate of change of its $y$-coordinate is $\frac{dy}{dt} = -1$. Find the rate of change of the distance $r$ of this moving point from the origin, when it has coordinates $(1, 3)$.

(Hint: Write down the relation between the distance $r$ from the origin and the $x$ and $y$ coordinates of the point. Then use the Related Rates technique to get a relation between the rate of change of the distance $\frac{dr}{dt}$ and $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

\[ r = \sqrt{x^2 + y^2} \]

\[ \Rightarrow 4 \frac{dx}{dt} = \frac{1}{2} (x^2 + y^2)^{-\frac{1}{2}} \left( 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \right) \]

At \( x = 1, \ y = 3, \ \frac{dx}{dt} = 2, \ \frac{dy}{dt} = -1 \)

\[ \Rightarrow \frac{dx}{dt} = \frac{1}{2} \left( 1^2 + 3^2 \right)^{-\frac{1}{2}} \left( 2(1)(2) + 2(3)(-1) \right) \]

\[ = \frac{1}{2} \cdot 10^{-\frac{1}{2}} \cdot (4 - 6) = \left[ -\frac{1}{\sqrt{10}} \right] \]

Both are OK.