

No notes or calculators. You can leave an answer as a numerical expression without computing the final value. For example, this is a perfectly acceptable answer :
 $((250 - 63)/(1 - e^{(-6 \cdot 3.5)})) * \ln(27/168)$. Show your work clearly !!

1. (5 points) Use partial fractions to find a solution to the differential equation

$$\frac{dy}{dx} = (y-2)(y+2)$$

with the initial value condition that $y = 0$ when $x = 0$.

$$\begin{aligned} \text{Left: } \int \frac{1}{(y-2)(y+2)} dy &= \int dx \\ \frac{1}{(y-2)(y+2)} &= \frac{A}{y-2} + \frac{B}{y+2} \\ &= \frac{A(y+2) + B(y-2)}{(y-2)(y+2)} \\ \therefore A+B &= 0, \quad 2A-2B = 1 \\ \therefore A &= \frac{1}{4}, \quad B = -\frac{1}{4} \end{aligned}$$

$$\begin{aligned} \int \frac{\frac{1}{4}}{y-2} + \frac{-\frac{1}{4}}{y+2} dy &= \int dx \\ \Rightarrow \frac{1}{4} \ln|y-2| - \frac{1}{4} \ln|y+2| &= x + C \\ \Rightarrow \frac{1}{4} \ln \frac{|y-2|}{|y+2|} &= x + C \Rightarrow \ln \frac{|y-2|}{|y+2|} = 4x + 4C \\ \Rightarrow \frac{|y-2|}{|y+2|} &= e^{4x+4C} \Rightarrow \frac{y-2}{y+2} = \pm e^{4C} e^{4x} \end{aligned}$$

2. (5 points) Find the equilibrium points for the differential equation

$$\frac{dy}{dx} = y^3 - 3y^2 + 2y$$

and discuss the stability of the largest equilibrium point.

$$\text{Setting } y^3 - 3y^2 + 2y = 0$$

$$\Rightarrow y(y^2 - 3y + 2) = 0$$

$$\Rightarrow y(y-1)(y-2) = 0$$

$$\Rightarrow y = 0, 1, 2$$

Largest equilibrium value = 2.

$$y = \frac{2 - 2e^{4x}}{1 + e^{4x}}$$

1

$$\Rightarrow y = \frac{2 + 2C_1 e^{4x}}{1 - C_1 e^{4x}}$$

Using $y=0$ when $x=0$

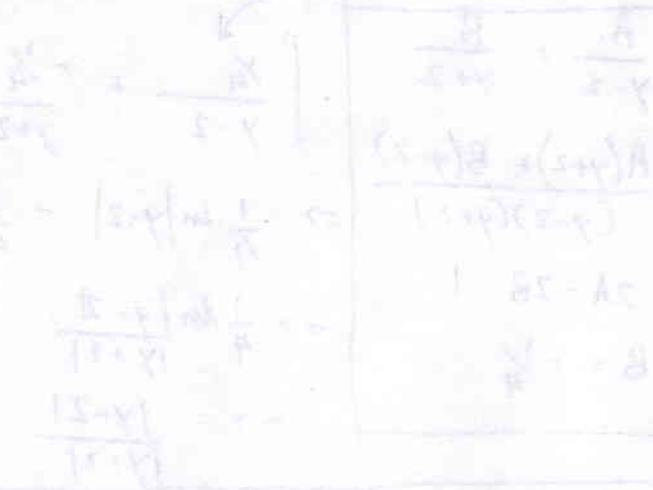
$$0 = \frac{2 + 2C_1}{1 - C_1} \Rightarrow C_1 = -1$$

$$\textcircled{*} \frac{d}{dy}(y^3 - 3y^2 + 2y) = 3y^2 - 6y + 2$$

At $y=2$

we get $3(2)^2 - 6(2) + 2$
 $= 2 > 0$

\Rightarrow Unstable equilibrium



$$y_1 + y_2 - y_0 = \frac{2}{3}$$

$$0 = 2x - \frac{2}{3}$$

$$0 = (2x - \frac{2}{3})x$$

$$\therefore 0 = (2x)(1 - \frac{1}{3}x)$$

$$2x(1 - \frac{1}{3}x) = x$$

∴ $x = 0$ or $x = \frac{3}{2}$

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| $\frac{25}{9} - \frac{5}{3} = \frac{5}{9}$ |
| $\frac{5}{9} + 1 = \frac{14}{9}$ |