Math 17B - UCD Quiz 6 Name

No notes or calculators. You can leave an answer as a numerical expression without computing the final value. For example, this is a perfectly acceptable answer : $((250-63)/(1-e^{(-6*3.5)}))*ln(27/168)$. Show your work clearly !!

1. (5 points) Use partial fractions to find a solution to the differential equation

$$\frac{dy}{dx} = (y-2)(y+2)$$

with the initial value condition that $y = 0$ when $x = 0$.

$$\int \frac{1}{(y-2)(y+2)}\,dy = \int dx$$

$$\frac{1}{(y-2)(y+2)} = \frac{A}{y-2} + \frac{B}{y+2} = \frac{A(y+2)+B(y-2)}{(y-2)(y+2)}$$

$\therefore\ A+B=0,\ 2A-2B=1$

$\therefore\ A = \frac{1}{4},\ B = -\frac{1}{4}$

$$\int \frac{1/4}{y-2} + \frac{-1/4}{y+2}\,dy = \int dx$$

$$\Rightarrow \frac{1}{4}\ln|y-2| - \frac{1}{4}\ln|y+2| = x + C$$

$$\Rightarrow \frac{1}{4}\ln\frac{|y-2|}{|y+2|} = x+C \Rightarrow \ln\frac{|y-2|}{|y+2|} = 4x+4C$$

$$\Rightarrow \frac{|y-2|}{|y+2|} = e^{4x+4C} \Rightarrow \frac{y-2}{y+2} = \pm e^{4C}e^{4x}$$

$$\Rightarrow \frac{y-2}{y+2} = C_1 e^{4x}$$

$$\Rightarrow (y-2) = C_1 e^{4x}(y+2)$$

$$\Rightarrow y-2 = yC_1e^{4x} + 2C_1e^{4x}$$

$$\Rightarrow y(1-C_1e^{4x}) = 2 + 2C_1e^{4x}$$

$$\Rightarrow y = \frac{2+2C_1e^{4x}}{1-C_1e^{4x}}$$

Using $y=0$ when $x=0$

$$0 = \frac{2+2C_1}{1-C_1} \Rightarrow \boxed{C_1=-1}$$

$$\boxed{y = \frac{2-2e^{4x}}{1+e^{4x}}}$$

2. (5 points) Find the equilibrium points for the differential equation

$$\frac{dy}{dx} = y^3 - 3y^2 + 2y$$

and discuss the stability of the *largest* equilibrium point

Setting $y^3 - 3y^2 + 2y = 0$

$$\Rightarrow y(y^2-3y+2) = 0$$

$$\Rightarrow y(y-1)(y-2) = 0$$

$$\Rightarrow y = 0, 1, 2$$

Largest equilibrium value $= 2$.

$$\frac{d}{dy}\left(y^3 - 3y^2 + 2y\right) = 3y^2 - 6y + 2$$

At $\underline{y = 2}$

we get $3(2)^2 - 6(2) + 2$

$= 2 > 0$

$\Rightarrow$ **Unstable equilibrium**