1. **(10 pts)** For each of the following pair of statements, circle the correct statement :

(a) $4^x/4^y = 4^{x-y}$ $\qquad$ $4^x/4^y = 4^{x/y}$.

(b) $\int \frac{f(x)}{g(x)}dx = \frac{\int f(x)dx}{\int g(x)dx}$ $\qquad$ $\int [f(x) + g(x)] = \int f(x) + \int g(x)$.

(c) $log_7 x = \frac{ln(x)}{ln(7)}$ $\qquad$ $log_7 x = ln(7)ln(x)$.

(d) $(lnx)^5 = 5ln(x)$ $\qquad$ $ln(x^5) = 5ln(x)$.

(e) $ln(x) + ln(y) = ln(x+y)$ $\qquad$ $ln(x) + ln(y) = ln(xy)$.

2. **(20 pts)** Differentiate the following functions.

(a) $\frac{e^x+e^{-x}}{2}$.

$$\frac{d}{dx}\left(\frac{e^x + e^{-x}}{2}\right) = \frac{1}{2}\frac{d}{dx}(e^x + e^{-x})$$

$$= \boxed{\frac{1}{2}\left[e^x - e^{-x}\right]}$$

(b) $ln(x\sqrt{4+x^2})$. $= ln(x) + \frac{1}{2}ln(4+x^2)$

$$\frac{d}{dx}\left(ln(x) + \frac{1}{2}ln(4+x^2)\right) = \frac{1}{x} + \frac{1}{2}\cdot\frac{1}{4+x^2}(2x)$$

$$= \boxed{\frac{1}{x} + \frac{x}{4+x^2}}$$

(c) $(e^x + x^2)^2$.

$$\frac{d}{dx}(e^x + x^2)^2 \underset{\text{Rule}}{\overset{\text{chain}}{=}} 2(e^x + x^2)\frac{d}{dx}(e^x + x^2)$$

$$\boxed{= 2(e^x + x^2)(e^x + 2x)}$$

(d) $x2^x$.

$$\frac{d}{dx}\left(x2^x\right) = 2^x\frac{d}{dx}(x) + x\frac{d}{dx}\left(2^x\right)$$

$$= \boxed{2^x + x(\ln 2)2^x}$$

(e) $(x+1)^{5x}$.

Rewrite $(x+1)^{5x} = e^{\ln\left((x+1)^{5x}\right)}$

$$= e^{5x\ln(x+1)}$$

$$\frac{d}{dx}\left(e^{5x\ln(x+1)}\right) \underset{\text{Rule}}{\overset{\text{Chain}}{=}} e^{5x\ln(x+1)}\frac{d}{dx}\left(5x\ln(x+1)\right)$$

$$= e^{5x\ln(x+1)}\left[5x\frac{d}{dx}\left(\ln(x+1)\right) + \ln(x+1)\frac{d}{dx}(5x)\right]$$

$$= e^{5x\ln(x+1)}\left[\frac{5x}{x+1} + \ln(x+1)\cdot 5\right]$$

$$= \boxed{(x+1)^{5x}\left[\frac{5x}{x+1} + 5\ln(x+1)\right]}$$

3. A bank offers 5% yearly rate of interest.

(a) **(3 pts)** I deposit \$1000 at the beginning of the year. The interest in the bank is compounded yearly. How much money do I have at the end of 5 years ?

Use formula $A = P(1+r)^t$

$$= 1000(1+0.05)^5$$

$$= \boxed{1000(1.05)^5}$$

You can leave your answer like this!

(b) **(3 pts)** Now suppose the interest in the bank is compounded every 6 months, that is, it is compounded 2 times a year. If I deposit \$1000 at the beginning of the year, how much will I have after 5 years ?

Use formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ where $n=2$.

$$1000\left(1+\frac{0.05}{2}\right)^{2\times 5}$$

$$= \boxed{1000(1.025)^{10}}$$

(c) **(4 pts)** Now suppose the interest in the bank is compounded continuously. If I deposit \$1000 at the beginning of the year, how much time does it take for the amount to become \$3000 ?

Use formula $A = Pe^{rt}$

$$3000 = 1000e^{0.05t}$$

$$\Rightarrow \frac{3000}{1000} = e^{0.05t}$$

$$\Rightarrow 3 = e^{0.05t}$$

$$\Rightarrow \ln 3 = 0.05t \Rightarrow t = \boxed{\frac{\ln 3}{0.05}}$$

4. **(10 pts)** Consider the exponential growth function

$$y = Ce^{kt}.$$

Suppose we know that $y = 5$ when $t = 1$, and $y = 10$ when $t = 2$. Compute the values of C and k.

$$y = 5, \quad t = 1$$

$$\Rightarrow \quad 5 = Ce^{k \cdot 1}$$

$$\Rightarrow \quad \frac{5}{e^k} = C \quad \text{---} \quad (*)$$

Also $\quad y = 10, \quad t = 2$

$$\Rightarrow \quad 10 = Ce^{k \cdot 2}$$

Substitute C using $(*)$

$$10 = \frac{5}{e^k} \cdot e^{2k}$$

$$\Rightarrow \quad 10 = 5e^k$$

$$\Rightarrow \quad 2 = e^k$$

$$\Rightarrow \quad k = \ln 2$$

$$\therefore \quad C = \frac{5}{e^k} = \frac{5}{e^{\ln 2}} = \frac{5}{2} = \boxed{2.5}$$

5. **(30 pts)** Compute the following integrals.

(a) $\int \frac{(ln(x))^2}{x} dx$

Substitute $u = (\ln x)$

$\frac{dy}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x} dx$

So $\int \frac{(\ln x)^2}{x} dx = \int u^2 du = \frac{u^3}{3} + C = \boxed{\frac{(\ln x)^3}{3} + C}$

(b) $\int \frac{x^2+5}{\sqrt{x}} dx$

$$= \int \left(\frac{x^2}{\sqrt{x}} + \frac{5}{\sqrt{x}}\right) dx = \int \left(x^{3/2} + 5x^{-1/2}\right) dx$$

$$= \int x^{3/2} dx + \int 5x^{-1/2} dx$$

$$= \frac{x^{3/2+1}}{3/2+1} + 5\frac{x^{(-1/2+1)}}{-1/2+1} + C$$

$$= \boxed{\frac{2x^{5/2}}{5} + 10x^{1/2} + C}$$

(c) $\int x^2 e^{x^3} dx$

Substitute $u = x^3$

$\frac{du}{dx} = 3x^2 \Rightarrow du = 3x^2 dx$

So $\int x^2 e^{x^3} dx = \int \left(\frac{1}{3} \cdot 3\right) x^2 e^{x^3} dx = \int \frac{1}{3} e^{x^3} 3x^2 dx$

Replace $3x^2 dx = du$ & $x^3 = u$

$$\int \frac{1}{3} e^u du = \frac{1}{3} e^u + C = \boxed{\frac{1}{3} e^{x^3} + C}$$

(d) $\int \frac{x^4+4x^3+3}{x+4}dx$

Use long division to rewrite $\dfrac{x^4+4x^3+3}{x+4} = x^3 + \dfrac{3}{x+4}$

So $\displaystyle\int \frac{x^4+4x^3+3}{x+4}dx = \int\left(x^3 + \frac{3}{x+4}\right)dx = \int x^3 dx + \int \frac{3}{x+4}dx$

Now for $\displaystyle\int \frac{3}{x+4}dx$ use substitution $u = x+4$, $\dfrac{du}{dx} = 1 \Rightarrow du = dx$

$\therefore \displaystyle\int \frac{3}{x+4}dx$

$= \displaystyle\int \frac{3}{u}du$

$= 3\ln|u| + C$

$= 3\ln|x+4| + C$

So final integral is

$\displaystyle\int x^3 dx + \int \frac{3}{x+4}dx$

$= \boxed{\dfrac{x^4}{4} + 3\ln|x+4| + C}$

(e) $\int \frac{e^{-x}}{1-e^{-x}}dx$

Substitute $1-e^{-x} = u$

$\dfrac{du}{dx} = \dfrac{d}{dx}(1-e^{-x}) = 0 - \dfrac{d}{dx}(e^{-x})$

$= e^{-x}$

$\Rightarrow du = e^{-x}dx$

So $\displaystyle\int \frac{e^{-x}}{1-e^{-x}}dx = \int \frac{1}{u}du$

$= \ln|u| + C$

$= \boxed{\ln|1-e^{-x}| + C}$