HW 9 Solutions MAT 17B

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5.

$$\frac{2x-3}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$= \frac{A(x+1) + Bx}{x(x+1)}$$

$$= \frac{(A+B)x + A}{x(x+1)}$$

$\therefore \quad A+B = 2, \quad A = -3$

$\Rightarrow \quad B = 5$

$$\therefore \quad \frac{2x-3}{x(x+1)} = \frac{-3}{x} + \frac{5}{x+1}$$

6.

$$-\frac{x+1}{(2x-1)(x-1)} = -\left(\frac{A}{2x-1} + \frac{B}{x-1}\right)$$

$$= -\left(\frac{A(x-1) + B(2x-1)}{(2x-1)(x-1)}\right)$$

$$= -\left(\frac{(A+2B)x + (-A-B)}{(2x-1)(x-1)}\right)$$

$\Rightarrow \quad A+2B = 1, \quad -A-B = 1$

solving for A, B : $A = -3, B = 2$

$$\therefore \quad -\frac{x+1}{(2x-1)(x-1)} = -\left(\frac{-3}{2x-1} + \frac{2}{x-1}\right)$$

$$= \frac{3}{2x-1} - \frac{2}{x-1}$$

9. $$\frac{5x-1}{x^2-1} = \frac{5x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$= \frac{A(x+1) + B(x-1)}{(x-1)(x+1)}$$

$$= \frac{(A+B)x + (A-B)}{(x-1)(x+1)}$$

$\therefore \quad A+B = 5, \quad A-B = -1$

solving for A, B: $A = 2$, $B = 3$

$$\therefore \quad \frac{5x-1}{x^2-1} = \frac{2}{x-1} + \frac{3}{x+1}$$

10. $$\frac{9x-7}{2x^2-7x+3} = \frac{9x-7}{(2x-1)(x-3)} = \frac{A}{2x-1} + \frac{B}{x-3}$$

$$= \frac{A(x-3) + B(2x-1)}{(2x-1)(x-3)}$$

$$= \frac{(A+2B)x + (-3A-B)}{(x-3)(2x-1)}$$

$\therefore \quad A+2B = 9, \quad -3A-B = -7$

solving for A, B: $A = 1$, $B = 4$.

$$\frac{9x-7}{2x^2-7x+3} = \frac{1}{2x-1} + \frac{4}{x-3}$$

11. $\dfrac{4x+1}{x^2-3x-10} = \dfrac{4x+1}{(x-5)(x+2)} = \dfrac{A}{x-5} + \dfrac{B}{x+2}$

$$= \frac{A(x+2) + B(x-5)}{(x-5)(x+2)}$$

$$= \frac{(A+B)x + (2A-5B)}{(x-5)(x+2)}$$

$\therefore \quad A+B = 4 \quad , \quad 2A-5B = 1$

$\therefore \quad A = 3 \quad , \quad B = 1$

$$\frac{4x+1}{x^2-3x-10} = \frac{3}{x-5} + \frac{1}{x+2}.$$

13. $\displaystyle\int \frac{1}{x(x-2)}\,dx$. $\qquad \dfrac{1}{x(x-2)} = \dfrac{A}{x} + \dfrac{B}{x-2}$

$$= \frac{A(x-2) + Bx}{x(x-2)}$$

$$= \frac{(A+B)x - 2A}{x(x-2)}$$

$A+B = 0 \quad , \quad -2A = 1$

$\Rightarrow \quad A = -\frac{1}{2} \quad , \quad B = \frac{1}{2}$

$$\therefore \int \frac{1}{x(x-2)}\,dx = \int \left(\frac{-1}{2x} + \frac{1}{2(x-2)}\right)dx$$

$$= \boxed{-\frac{1}{2}\ln|x| + \frac{1}{2}\ln|x-2| + C}$$

15. $\displaystyle\int \frac{1}{(x+1)(x-3)}\,dx$

$$\frac{1}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

get $A = -\frac{1}{4}$, $B = +\frac{1}{4}$

$$\therefore \int \frac{1}{(x+1)(x-3)}\,dx = -\frac{1}{4}\ln|x+1| + \frac{1}{4}\ln|x-3| + C$$

16. $\displaystyle\int \frac{1}{(x-1)(x+2)}\,dx$

Use partial fractions to get.

$$\frac{1}{(x-1)(x+2)} = \frac{1}{3(x-1)} - \frac{1}{3(x+2)}$$

$$\therefore \int \frac{1}{(x-1)(x+2)}\,dx = \boxed{\frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + C}$$

23. $\displaystyle\int \frac{1}{x^2-2x+2}\,dx = \int \frac{1}{(x-1)^2+1}\,dx$

Substitute $u = x-1$

$$\frac{du}{dx} = 1 \implies du = dx$$

$$\therefore \int \frac{1}{(x-1)^2+1}\,dx = \int \frac{1}{u^2+1}\,du = \tan^{-1} u$$

$$\therefore \int \frac{1}{x^2-2x+2}\,dx = \boxed{\tan^{-1}(x-1) + C}$$

29. $\displaystyle\int \frac{1}{x^2-9}\,dx = \int \frac{1}{(x-3)(x+3)}\,dx$

Using partial fractions, rewrite.

$$\int \frac{1}{(x-3)(x+3)}\,dx = \int \frac{1}{6(x-3)} - \frac{1}{6(x+3)}\,dx$$

$$= \boxed{\frac{1}{6}\ln|x-3| - \frac{1}{6}\ln|x+3| + C}$$

30. $\int \frac{1}{x^2+9}\,dx$

$= \frac{1}{9}\int \frac{1}{\left(\frac{x}{3}\right)^2+1}\,dx$

Substitute $u = \frac{x}{3}$

$du = \frac{1}{3}dx \Rightarrow 3du = dx$

$\therefore \int \frac{1}{x^2+9} = \frac{1}{9}\int \frac{1}{u^2+1}(3du)$

$= \frac{1}{3}\tan^{-1} u = \boxed{\frac{1}{3}\tan^{-1}\frac{x}{3} + C}$

31. $\int \frac{1}{x^2-x-2}\,dx = \int \frac{1}{(x-2)(x+1)}\,dx$

Using partial fractions, rewrite

$\int \frac{1}{(x-2)(x+1)}\,dx = \int \left(\frac{1}{3(x-2)} - \frac{1}{3(x+1)}\right)dx$

$= \boxed{\frac{1}{3}\ln|x-2| - \frac{1}{3}\ln|x+1| + C}$

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1. $$\frac{2x^2+5x-1}{x+2}$$

$$
\begin{array}{r|l}
 & \quad 2x \; + 1 \\ \hline
x+2 & 2x^2 + 5x - 1 \\
 & -\,2x^2 \overset{(-)}{+} 4x \\ \cline{2-2}
 & \qquad\quad x - 1 \\
 & \qquad (-)\,x \overset{(-)}{+} 2 \\ \cline{2-2}
 & \qquad\qquad -3
\end{array}
$$

$$\therefore \quad \frac{2x^2+5x-1}{x+2} = \boxed{2x+1 + \frac{(-3)}{x+2}}$$

2. $$\frac{x^2-4x-1}{x-1}$$

$$
\begin{array}{r|l}
 & \quad x \; - 3 \\ \hline
x-1 & x^2 - 4x - 1 \\
 & (-)\,x^2 \overset{(+)}{-} x \\ \cline{2-2}
 & \qquad -3x - 1 \\
 & \quad (+)\,3x \overset{(-)}{+} 3 \\ \cline{2-2}
 & \qquad\qquad -4
\end{array}
$$

$$\therefore \quad \frac{x^2-4x-1}{x-1} = \boxed{x-3 + \frac{-4}{x-1}}$$

4. $\dfrac{x^3 - 3x^2 - 15}{x^2 + x + 3}$

$$
\begin{array}{r|l}
 & \quad x - 4 \\
\hline
x^2 + x + 3 & x^3 - 3x^2 - 15 \\
 & \underline{(-)\,x^3 \,(\mp)\, x^2 \,(\mp)\, 3x} \\
 & \qquad -4x^2 - 3x - 15 \\
 & \qquad \underline{(\pm)\,4x^2 \,(\pm)\, 4x \,(\pm)\, 12} \\
 & \qquad\qquad x - 3
\end{array}
$$

$\therefore \quad \dfrac{x^3 - 3x^2 - 15}{x^2 + x + 3} = x - 4 + \dfrac{x - 3}{x^2 + x + 3}$

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17. $\displaystyle\int \frac{x^2 - 2x - 2}{x^2(x+2)}\,dx$

Proper rational function with repeated linear term x^2 in denominator.

$\therefore$ Partial fraction decomposition:

$$\frac{x^2 - 2x - 2}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$$

$$= \frac{Ax + B + Cx^2}{x^2(x+2)}$$

$\Rightarrow \quad C = 1, \quad A = -2, \quad B = -2$

$$\therefore \int \frac{x^2 - 2x - 2}{x^2(x+2)} = \int \left(\frac{-2}{x} + \frac{-2}{x^2} + \frac{1}{x+2} \right) dx$$

$$= \boxed{-2\ln|x| + \frac{2}{x} + \ln|x+2| + C}$$

17. $\displaystyle\int \frac{x^2-2x-2}{x^2(x+2)}\,dx$

Proper rational function with repeated linear terms x^2 in the denominator.

$\therefore$ Partial fraction decomposition:

$$\frac{x^2-2x-2}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+2)}$$

$$= \frac{A\,x(x+2)+B(x+2)+Cx^2}{x^2(x+2)}$$

$$= \frac{(A+C)x^2+(2A+B)x+2B}{x^2(x+2)}$$

$\Rightarrow \quad A+C=1, \quad 2A+B=-2, \quad 2B=-2$

$\Rightarrow B=-1$

$\Rightarrow A=-1/2$

$\Rightarrow C=3/2$

$$\therefore \int \frac{x^2-2x-2}{x^2(x+2)} = \int \frac{(-1/2)}{x} + \frac{-1}{x^2} + \frac{3/2}{x+2}\,dx$$

$$= \boxed{-\frac{1}{2}\ln|x| + \frac{1}{x} + \frac{3}{2}\ln|x+2| + C}$$

18. $\displaystyle\int \frac{4x^2-x-1}{(x+1)^2(x-3)}\,dx$

Repeated linear term $(x+1)^2$

$$\frac{4x^2-x-1}{(x+1)^2(x-3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-3}$$

$$= \frac{A(x+1)(x-3) + B(x-3) + C(x+1)^2}{(x+1)^2(x-3)}$$

$$= \frac{(A+C)x^2 + (-2A+2C+B)x + (-3A-3B+C)}{(x+1)^2(x-3)}$$

$\Rightarrow A+C = 4, \quad -2A+2C+B = -1, \quad -3A-3B+C = -1$

$\Rightarrow C = 4-A$ Plugging into other 2:

$$-2A + 2(4-A) + B = -1$$
$$-3A - 3B + (4-A) = -1$$

Simplifying:

$$-4A + B = -9$$
$$-4A - 3B = -5$$

$\Rightarrow B = -1, \quad A = 2$

$\Rightarrow C = 2$

$$\therefore \int \frac{4x^2-x-1}{(x+1)^2(x-3)} = \int \frac{2}{x+1} + \frac{-1}{(x+1)^2} + \frac{2}{x-3}\,dx$$

$$= \boxed{2\ln|x+1| + \frac{1}{x+1} + 2\ln|x-3| + C}$$

$$\cancel{2 \int \frac{x^3 - x^2 + x - 4}{(x^2+1)(x^2+4)}}$$

33.
$$\int \frac{x^2+1}{x^2+3x+2}\,dx$$

Improper Rational function → so use long division first

$$\frac{x^2+1}{x^2+3x+2} = 1 + \frac{(-3x-1)}{x^2+3x+2}$$

$$\therefore \int \frac{x^2+1}{x^2+3x+2}\,dx = \int 1 + \frac{(-3x-1)}{x^2+3x+2}\,dx$$

$$= x + \int \frac{-3x-1}{x^2+3x+2}\,dx$$

Proper rational function & $x^2+3x+2 = (x+2)(x+1)$ so denominator can be factored into linear terms.

$$\therefore \frac{-3x-1}{x^2+3x+2} = \frac{A}{x+2} + \frac{B}{x+1}$$

$$= \frac{A(x+1) + B(x+2)}{(x+2)(x+1)}$$

$$= \frac{(A+B)x + (A+2B)}{(x+2)(x+1)}$$

$\therefore \quad A + B = -3 \quad , \quad A + 2B = -1$

$$\Rightarrow \boxed{B = 2 \quad , \quad A = -5}$$

$$\therefore \int \frac{-3x-1}{x^2+3x+2}\,dx = \int \left(\frac{-5}{x+2} + \frac{2}{x+1} \right) dx$$

$$= -5\ln|x+2| + 2\ln|x+1|$$

$$\therefore \int \frac{x^2+1}{x^2+3x+2} = \boxed{x - 5\ln|x+2| + 2\ln|x+1| + C}$$

35. $\displaystyle\int \frac{x^2+4}{x^2-4}\,dx$

Use long division : $\displaystyle\frac{x^2+4}{x^2-4} = 1 + \frac{8}{x^2-4}$

$$\therefore \int \frac{x^2+4}{x^2-4}\,dx = \int\left(1+\frac{8}{x^2-4}\right)dx = x + \underbrace{\int \frac{8}{x^2-4}\,dx}$$

denominator can be factored $x^2-4 = (x-2)(x+2)$

Partial fractions:

$$\frac{8}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$= \frac{A(x+2) + B(x-2)}{(x-2)(x+2)}$$

$$= \frac{(A+B)x + (2A-2B)}{(x-2)(x+2)}$$

$$\Rightarrow \quad A+B=0, \quad 2A-2B=8$$

$$A=2, \quad B=-2$$

$$\therefore \int \frac{8}{x^2-4}\,dx = \int \frac{2}{x-2} + \frac{-2}{x+2}\,dx = 2\ln|x-2| - 2\ln|x+2|$$

$$\therefore \int \frac{x^2+4}{x^2-4}\,dx = \boxed{x + 2\ln|x-2| - 2\ln|x+2| + C}$$

3b. $\displaystyle\int \frac{x^4+3}{x^2-4x+3}\,dx$

Use long division

$$\begin{array}{r|l} & x^2 + 4x + 13 \\ \hline x^2-4x+3 & x^4 + 3 \\ & (-)x^4 (\pm) 4x^3 (\mp) 3x^2 \\ \hline & +4x^3 - 3x^2 + 3 \\ & (\mp)4x^3 (\pm) 16x^2 (\mp) 12x \\ \hline & 13x^2 - 12x + 3 \\ & (-)13x^2 (\pm) 52x (\mp) 39 \\ \hline & 40x - 36 \end{array}$$

$$\therefore \int \frac{x^4+3}{x^2-4x+3}\,dx = \int (x^2+4x+13) + \frac{40x-36}{x^2-4x+3}\,dx$$

$$= \frac{x^3}{3} + 2x^2 + 13x + \int \frac{40x-36}{x^2-4x+3}\,dx$$

For $\displaystyle\int \frac{40x-36}{x^2-4x+3}$: denominator factors : $(x-3)(x-1)$

Use partial fractions :

$$\frac{40x-36}{x^2-4x+3} = \frac{A}{x-3} + \frac{B}{x-1}$$

$$= \frac{A(x-1) + B(x-3)}{(x-1)(x-3)}$$

$$= \frac{(A+B)x + (-A-3B)}{(x-1)(x-3)}$$

$\Rightarrow \quad A+B = 40 \quad , \quad -A-3B = -36$

$B = -2 \quad , \quad A = 42$

$$\therefore \int \frac{40x-36}{x^2-4x+3} = \int \frac{42}{x-3} + \frac{-2}{x-1}\,dx$$

$$= \boxed{42\ln|x-3| + -2\ln|x-1| + C}$$

$$\therefore \int \frac{x^4+3}{x^2-4x+3} = \boxed{\frac{x^3}{3} + 2x^2 + 13x + 42\ln|x-3| - 2\ln|x-1| + C}$$

38. $\displaystyle\int_3^5 \frac{x}{x-1}\,dx$

Compute $\displaystyle\int \frac{x}{x-1}\,dx$

Use substitution $u = x-1$

$\dfrac{du}{dx} = 1 \Rightarrow du = dx$

$$\therefore \int \frac{x}{x-1}\,dx = \int \frac{u+1}{u}\,du = \int 1 + \frac{1}{u}\,du$$

$$= u + \ln|u|$$

$$\therefore \int \frac{x}{x-1}\,dx = (x-1) + \ln|x-1|$$

$$\therefore \int_3^5 \frac{x}{x-1}\,dx = \big((5-1) + \ln|5-1|\big) - \big((3-1) + \ln|3-1|\big)$$

$$= \boxed{2 + \ln 4 - \ln 2}$$

50. $\displaystyle\int \frac{1}{(x^2-x-2)^2}\,dx$

Denominator can be factored:

$$x^2-x-2 = (x-2)(x+1)$$

$$\therefore \frac{1}{(x^2-x-2)^2} = \frac{1}{((x-2)(x+1))^2} = \frac{1}{(x-2)^2(x+1)^2}$$

Repeated linear terms:

$$\frac{1}{(x-2)^2(x+1)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x+1)} + \frac{D}{(x+1)^2}$$

$$= \frac{A(x-2)(x+1)^2 + B(x+1)^2 + C(x-2)^2(x+1) + D(x-2)^2}{(x-2)^2(x+1)^2}$$

$$= \frac{A(x-2)(x^2+2x+1) + B(x^2+2x+1) + C(x^2-4x+4)(x+1) + D(x^2-4x+4)}{(x-2)^2(x+1)^2}$$

$$= \frac{(A+C)x^3 + (B-3C+D)x^2 + (-3A+2B-4D)x + (-2A+B+4C+4D)}{(X-2)^2(x+1)^2}$$

$\therefore$

$$A + C = 0$$

$$B - 3C + D = 0$$

$$-3A + 2B - 4D = 0$$

$$-2A + B + 4C + 4D = 1$$

use $C = -A$ in the other 3 equations:

$$\left.\begin{aligned} &B - 3(-A) + D = 0 \\ &-3A + 2B - 4D = 0 \\ &-2A + B + 4(-A) + 4D = 1 \end{aligned}\right\} \quad \begin{aligned} &3A + B + D = 0 \\ &-3A + 2B - 4D = 0 \\ &-6A + B + 4D = 1 \end{aligned}$$

from 1st equation

$$D = -B - 3A$$

Plugging into the next 2:

$$-3A + 2B - 4(-B - 3A) = 0$$

$$-6A + B + 4(-B - 3A) = 1$$

Simplifying:

$$\left.\begin{aligned} 9A - 2B &= 0 \\ -18A - 3B &= 1 \end{aligned}\right\} \Rightarrow \begin{aligned} B &= -1 \\ A &= 2/9 \end{aligned}$$

$$\Rightarrow D = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\Rightarrow C = -\frac{2}{9}$$

$$\therefore \int \frac{1}{(x^2 - x - 2)^2}\,dx = \int \frac{1}{(x-2)^2(x+1)^2}\,dx$$

$$= \int \frac{\left(\frac{2}{9}\right)}{(x-2)} + \frac{(-1)}{(x-2)^2} + \frac{\left(\frac{2}{9}\right)}{x+1} + \frac{\frac{1}{3}}{(x+1)^2}\,dx$$

$$= \boxed{\frac{2}{9}\ln|x-2| + \frac{1}{x-2} - \frac{2}{9}\ln|x+1| - \frac{1}{3(x+1)} + C}$$