HW 8 Solutions MAT 17B

Section 7.2 pg 343

40.

$$\int \sin \sqrt{x}\, dx$$

1. $u = \sqrt{x}$

2. $\frac{du}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow du = \frac{1}{2\sqrt{x}} dx$

Now $u = \sqrt{x}$

$\therefore\ 2u\,du = dx$

3. $\int \sin \sqrt{x}\, dx = \int \sin(u)(2u\,du)$

4. $\int (\sin u)(2u\,du) = 2\int u \sin u\, du$

Integration by parts

LATE Rule : $f(u) = u$, $g(u) = \sin u$

$\int g(u)\,du = \int (\sin u)\,du = -\cos u$

$\int f'(u) G(u)\,du = \int (-\cos u)\,du = -\sin u$

$\therefore\ 2\int u \sin u\, du = 2(-u\cos u - (-\sin u))$

$= -2u\cos u + 2\sin u$

$$\therefore \int \sin \sqrt{x}\, dx = \boxed{-2\sqrt{x} \cos \sqrt{x} + 2\sin \sqrt{x} + C}$$

41. $\int x^3 e^{-x^2/2}\,dx$

1. $u = -x^2/2$

2. $\frac{du}{dx} = -x \Rightarrow -du = x\,dx$

3. $\int x^3 e^{-x^2/2}\,dx = \int e^{-x^2/2} \cdot x^2 \cdot x\,dx$

$= \int e^u (-2u)(-du)$

$\left[\text{since } u = -\frac{x^2}{2} \quad \therefore -2u = x^2 \right]$

4. $\int e^u(-2u)(-du) = 2\int u e^u\,du$

Using Integration by parts.

$2\int u e^u\,du = 2(ue^u - e^u)$

5. $\int x^3 e^{-x^2/2} = 2\left(-\frac{x^2}{2} e^{-x^2/2} - e^{-x^2/2}\right)$

$= \boxed{-x^2 e^{-x^2/2} - e^{-x^2/2} + C}$

42. $\int x^5 e^{x^2} dx$

1. $u = x^2$

2. $\frac{du}{dx} = 2x \quad \Rightarrow \quad \frac{1}{2} du = x\,dx$

3. $\int x^5 e^{x^2} dx = \int e^{x^2} \cdot x^2 \cdot x^2 \cdot x\, dx$

$= \int e^u \cdot u \cdot u \cdot \left(\frac{1}{2} du\right)$

$= \frac{1}{2} \int u^2 e^u du$

4. $\frac{1}{2} \int u^2 e^u du$ Use integration by parts on $\int u^2 e^u du$

$f(u) = u^2$, $g(u) = e^u$

$\int g(u) du = e^u$, $\int f'(u) G(u) du$

$= \int 2u e^u du$

Need $\int 2u e^u du$

Use integration by parts again to get.

$\int 2u e^u du = 2(u e^u - e^u)$

$\therefore \frac{1}{2} \int u^2 e^u du = \frac{1}{2}\left(u^2 e^u - 2(u e^u - e^u)\right)$

$= \frac{1}{2} u^2 e^u - u e^u + e^u$

5. $\int x^5 e^{x^2} dx = \boxed{\frac{1}{2}(x^2)^2 e^{x^2} - x^2 e^{x^2} + e^{x^2} + C}$

44. $\int \sin x \cos^3 x \, e^{1-\sin^2 x} dx$

$= \int \sin x \cos^3 x \, e^{\cos^2 x} dx$ $\left(\overset{\text{since}}{\sin^2 x + \cos^2 x = 1} \right)$

1. Substitute $u = \cos^2 x$

2. $\frac{du}{dx} = 2\cos x(-\sin x)$ $\Rightarrow$ $-\frac{1}{2} du = \cos x \sin x \, dx$

3. $\int \sin x \cos^3 x \, e^{\cos^2 x} dx = \int e^{\cos^2 x} \cos^2 x \, \underbrace{\cos x \sin x \, dx}$

$$= \int e^u u \left(-\frac{1}{2} du\right)$$

$$= -\frac{1}{2} \int u e^u du$$

4. $-\frac{1}{2} \int u e^u du$ Use integration by parts to get

$$-\frac{1}{2} \int u e^u du = -\frac{1}{2}(u e^u - e^u)$$

5. $\int \sin x \cos^3 x \, e^{1-\sin^2 x} dx = \boxed{-\frac{1}{2}\left(\cos^2 x \, e^{\cos^2 x} - e^{\cos^2 x}\right) + C}$

47. $\int_1^4 \ln(\sqrt{x}+1)\,dx$

First evaluate $\int \ln(\sqrt{x}+1)\,dx$

1. Substitute $u = \sqrt{x}+1$

2. $\frac{du}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow du = \frac{1}{2\sqrt{x}}dx$

(since $u = \sqrt{x}+1 \Rightarrow u-1 = \sqrt{x}$)

$\therefore\ du = \frac{1}{2(u-1)}dx$

$\Rightarrow 2(u-1)du = dx$

3. $\int \ln(\sqrt{x}+1)dx = \int \ln(u)(2(u-1)\,du)$

$= 2\int (u-1)\ln u\,du$

4. Use integration by parts.

$f(u) = \ln u \quad g(u) = u-1$

$\int g(u)\,du = \int (u-1)du = \frac{u^2}{2} - u$

$\int f'(u)\,G(u)du = \int \frac{1}{u}\left(\frac{u^2}{2}-u\right)du$

$= \int \left(\frac{u}{2}-1\right)du$

$= \frac{u^2}{4} - u$

$\therefore\ 2\int (u-1)\ln u\,du = 2\left((\ln u)\left(\frac{u^2}{2}-u\right) - \left(\frac{u^2}{4}-u\right)\right)$

5. $\int \ln(\sqrt{x}+1)$

$$= \boxed{2\ln(\sqrt{x}+1)\left(\frac{(\sqrt{x}+1)^2}{2} - (\sqrt{x}+1)\right) - 2\left(\frac{(\sqrt{x}+1)^2}{4} - (\sqrt{x}+1)\right) + C}$$

$\therefore$ $\int_1^4 \ln(\sqrt{x}+1) = 2\ln(\sqrt{4}+1)\left(\frac{(\sqrt{4}+1)^2}{2} - (\sqrt{4}+1)\right)$

$$- 2\left(\frac{(\sqrt{4}+1)^2}{4} - (\sqrt{4}+1)\right)$$

$$- \left[2\ln(\sqrt{1}+1)\left(\frac{(\sqrt{1}+1)^2}{2} - (\sqrt{1}+1)\right) - 2\left(\frac{(\sqrt{1}+1)^2}{4} - (\sqrt{1}+1)\right)\right]$$

$$= 2(\ln 3)\left(\frac{3}{2}\right) + \frac{6}{4} + 2$$

70. $\int_0^{\pi/6} (1+\tan^2 x)\,dx$. Evaluate $\int (1+\tan^2 x)\,dx$

$$= \int \sec^2 x\,dx \quad \left(\begin{array}{c}1+\tan^2 x \\ =\sec^2 x\end{array}\right)$$

$$= \tan x$$

$$\therefore \int_0^{\pi/6} (1+\tan^2 x)\,dx = \boxed{\tan\frac{\pi}{6} - \tan 0}$$