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2. $\int 3x \cos x \, dx = 3 \int x \cos x \, dx$

Use integration by parts with

1st part $f(x) = x$ using LATE rule.
2nd part $g(x) = \cos x$

$\int x \cos x \, dx$

1. $\int g(x) dx = \int \cos x \, dx = \boxed{+\sin x} = G(x).$

2. $\int f'(x) G(x) dx = \int (1)(\sin x) dx = -\cos x$

3. $\int f(x) g(x) = f(x)G(x) - \int f'(x) G(x) dx$

$= x \sin x - (-\cos x) = \boxed{x \sin x + \cos x + C}$

7. $\int x e^x dx$: 1st part $f(x) = x$
2nd part $g(x) = e^x$

1. $\int g(x) dx = \int e^x dx = e^x = G(x)$

2. $\int f'(x) G(x) dx = \int (1) e^x dx = \int e^x dx = e^x$

3. $\int x e^x dx = \boxed{x e^x - e^x + C}$

10. $\int 2x^2 e^{-x}\,dx = 2\int x^2 e^{-x}\,dx$

To ~~evalute~~ evaluate $\int x^2 e^{-x}\,dx$, use integration by parts:

$$f(x) = x^2, \quad g(x) = e^{-x}$$

1. $\int g(x)\,dx = \int e^{-x}\,dx = -e^{-x}$

2. $\int f'(x)G(x)\,dx = \int (2x)(-e^{-x})\,dx$

$$= -2\int x e^{-x}\,dx$$

Use Integration by parts again:

$\int x e^{-x}\,dx$ $\quad f(x) = x, \quad g(x) = e^{-x}$

1. $\int g(x)\,dx = \int e^{-x}\,dx = -e^{-x}$

2. $\int f'(x)G(x)\,dx = \int (1)(e^{-x})\,dx$

$$= -(-e^{-x})$$

$$= e^{-x}$$

3. $\int x e^{-x} = x(-e^{-x}) - (e^{-x})$

$$= -e^{-x}(x+1)$$

$\therefore\ -2\int x e^{-x}\,dx = -2\left(-e^{-x}(x+1)\right) = 2e^{-x}(x+1)$

3. $\int x^2 e^{-x}\,dx = x^2(-e^{-x}) - \left(2e^{-x}(x+1)\right) = \boxed{-x^2e^{-x} - 2e^{-x}(x+1)}$

$$\therefore \int 2x^2 e^{-x}\,dx = \boxed{2\left(-x^2e^{-x} - 2e^{-x}(x+1)\right) + C}$$

(12) $\int x^2 \ln x \, dx$

Using LATE, $f(x) = \ln x$
$g(x) = x^2$

1. $\int g(x)\,dx = \int x^2 dx = \frac{x^3}{3}$

2. $\int f'(x)G(x)dx = \int \left(\frac{1}{x}\right)\left(\frac{x^3}{3}\right)dx = \int \frac{x^2}{3}dx$
$= \frac{x^3}{9}$

3. $\int x^2 \ln x \, dx = \boxed{\ln(x)\,\frac{x^3}{3} - \frac{x^3}{9} + C}$

(16) $\int x \csc^2 x \, dx$ Using LATE, $f(x) = x$
$g(x) = \csc^2(x)$

1. $\int g(x)dx = \int \csc^2 x \, dx = -\cot x$

2. $\int f'(x)\,G(x)dx = \int (1)(-\cot x)\,dx = -\int \cot x \, dx$
$= -(-\ln|\csc x|)$
$= \ln|\csc x|$

3. $\int x \csc^2 x \, dx = \boxed{x(-\cot x) - \ln|\csc x| + C}$

$\underline{22}$ $\int_1^4 \sqrt{x} \ln \sqrt{x}\, dx$

Using LATE : $f(x) = \ln \sqrt{x}$

$g(x) = \sqrt{x}$

1. $\int g(x)\,dx = \int \sqrt{x}\,dx = \frac{2}{3}x^{3/2}$

2. $\int f'(x)\,G(x)\,dx = \int \underbrace{\left(\frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}\right)}_{f'(x)} \frac{2}{3}x^{3/2}\,dx$

$= \int \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{\sqrt{x}} \cdot x^{3/2}\,dx$

$= \frac{1}{3}\int x^{1/2}\,dx = \frac{1}{3}\frac{2x^{3/2}}{3} = \frac{2}{9}x^{3/2}$

3. $\int \sqrt{x} \ln \sqrt{x}\,dx = (\ln \sqrt{x})\frac{2}{3}x^{3/2} - \frac{2}{9}x^{3/2}$

$\therefore \int_1^4 \sqrt{x} \ln \sqrt{x}\,dx = \left((\ln \sqrt{4})\frac{2}{3}(4)^{3/2} - \frac{2}{9}(4)^{3/2}\right)$

$- \left((\ln \sqrt{1})\frac{2}{3}(1)^{3/2} - \frac{2}{9}(1)^{3/2}\right)$

$= \frac{16\ln 2}{3} - \frac{8}{9} + \frac{2}{9} = \boxed{\frac{16\ln 2 - 2}{3}}$