Practice Midterm II
MAT-25 Advanced Calculus

Spring 2013

Name _____

- **This test is closed notes, closed book.**
- **You may use any definition/theorem/property discussed from class without proof - if you do, you must write the theorem statement in full detail.**
- **There are 10 pages and 7 questions total.**
- **The maximum score in the test is 85 points.**
- **IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM ANOTHER STUDENT'S EXAM. IT IS A VIOLATION OF THE UNIVERSITY'S HONOR CODE TO HAVE ANOTHER PERSON TAKE YOUR EXAM FOR YOU.**

Signature _____

Problem	Score	Max Possible
1		10
2		10
3		15
4		15
5		15
6		10
7		10
Total		85

Properties of absolute value function :

1. $|x| \geq 0 \quad \forall x \in \mathbb{R}$

2. $|x| < r$ if and only if $-r < x < r \quad \forall x, r \in \mathbb{R}$

3. $|-x| = |x| \quad \forall x \in \mathbb{R}$

4. $-|x| \leq x \leq |x| \quad \forall x \in \mathbb{R}$

5. $|xy| = |x||y| \quad \forall x, y \in \mathbb{R}$

6. $|x+y| \leq |x| + |y| \quad \forall x, y \in \mathbb{R}$

7. $|x| - |y| \leq |x-y| \quad \forall x, y \in \mathbb{R}$ and $|y| - |x| \leq |x-y| \quad \forall x, y \in \mathbb{R}$

You can use the following fact without proof :

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

1. (a) **(5 pts)** Give the precise definition of the limit of a sequence $\{s_n\}_{n=1}^{\infty}$.

(b)**(5 pts)** Explain the meaning of the sentence "$E \subseteq \mathbb{R}$ is dense in $\mathbb{R}$" in mathematical terms.

2. **(10 pts)** Let $f(x) = \frac{1}{x^2}$. Find the infimum of the set $E = \{f(x) : x \in \mathbb{R}\}$ and justify your answer.

3. **(15 pts)** Let $E \subseteq \mathbb{R}$ be any nonempty subset of real numbers that is bounded below. Let $L = \inf E$. Show that there exists a sequence $\{s_n\}_{n=1}^{\infty}$ such that $s_n \in E$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} s_n = L$.

4. **(15 pts)** Let $r \in \mathbb{R}$ be a fixed real number. Let $A \subseteq \mathbb{R}$ be a subset of real numbers. Let $B = \{x - r : x \in A\}$. Find a relation between $\sup(A)$ and $\sup(B)$, and prove it.

5. **(15 pts)** Let E be a subset of real numbers of the form

$$E = \{\frac{\sqrt{2}m}{n} : m \in \mathbb{Z}, n \in N\}.$$

Show that E is dense in $\mathbb{R}$. [Hint: Consider any interval (a, b). Manipulate this interval and use density of $\mathbb{Q}$]

6. **(10 pts)** Find

$$\lim_{n\to\infty} \frac{2n^2+1}{3n^2+2}$$

and justify your answer.

7. **(10 pts)** Show that $\min\{x, y\} = \frac{(x+y)}{2} - \frac{|x-y|}{2}$.