12.

2.2.6. Claim: $x_n = \sqrt{2}n$.

Pf: By induction of n.

Basis: $n=1$ $x_1 = \sqrt{2} = \sqrt{2}(1)$.

Induction Step: Assume $P(n-1)$, show $P(n)$

Assume. $x_{n-1} = \sqrt{2}(n-1)$.

$$x_n = \sqrt{2} + x_{n-1}$$
$$= \sqrt{2} + \sqrt{2}(n-1)$$
$$= \sqrt{2}n$$

Thus $x_n = \sqrt{2}n$ and $P(n)$ is true.

2.2.7. Claim: $x_n = (\sqrt{2})^n$

Pf: By induction on n.

Basis $n=1$. $x_1 = \sqrt{2} = (\sqrt{2})^1$

Induction Step: Assume $P(n-1)$, show $P(n)$.

Assume $x_{n-1} = (\sqrt{2})^{n-1}$

$$x_n = \sqrt{2}\,x_{n-1} = \sqrt{2}(\sqrt{2})^{n-1} = (\sqrt{2})^n$$

Thus $x_n = (\sqrt{2})^n$ and so $P(n)$ is true.

2.2.8. Claim: $x_n < 2$.

Pf: By induction on n.

Basis $n=1$. $x_1 = \sqrt{2} < 2$ ✓.

Induction Step : Assume $P(n-1)$, show $P(n)$.

Assume $x_{n-1} < 2$.

$$x_n = \sqrt{2+x_{n-1}}$$

$$< \sqrt{2+2} \quad \text{since } x_{n-1} < 2.$$

$$= 2.$$

$\therefore$ $x_n < 2$, and so $P(n)$ is true.

2.2.9. Claim : $x_n < x_{n+1}$

Basis : $x_1 = \sqrt{2} < \sqrt{2+\sqrt{2}} = x_2$

Induction step : Assume $P(n-1)$, show $P(n)$.

Assume $x_{n-1} < x_n$

$$\Rightarrow 2 + x_{n-1} < 2 + x_n$$

$$\Rightarrow \sqrt{2+x_{n-1}} < \sqrt{2+x_n}$$

$$\Rightarrow x_n < x_{n+1}$$

Thus, $P(n)$ is true.