Homework Problems : WEEK I

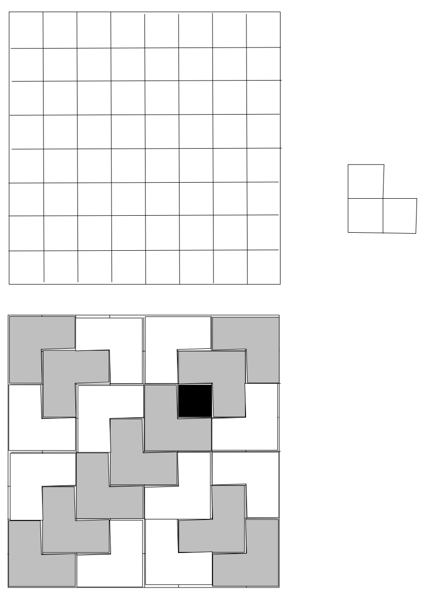
Problems to be handed in on Monday : A.6.1, A.8.2, Problems 2 and 5 below

1. Do A.5.1, A.5.2, A.6.1, A.7.3, A.8.1, A.8.2, A.8.4, A.8.5, A.8.9 (Carefully argue what is going wrong in A.8.9 - its tricky !).

2. Let x be a natural number. Prove that x^2 is even if and only if x is even. (This means you have to actually prove two statements, or two implications. Carefully write both with a separate proof for each).

3. Suppose you have a $2^n \times 2^n$ checkerboard, where n is a natural number - this means you have 2^n rows and 2^n columns. You are given an infinite supply of L-shaped dominoes of the form shown in the figure below. Show that for every natural number n, one can do the following. Mark *any* square in the $2^n \times 2^n$ checkerboard. Then one can tile the checkerboard using the L-shaped dominoes, such that the marked square is NOT covered, all other squares are covered, and no two dominoes overlap. (Hint : Induction !)

   An example with $n = 3$ is shown below. The black square is the marked square.



4. Suppose f is some function define over the interval $[0, 1]$. Show that if $\int_0^1 f(x)dx \neq 0$, then there is some $0 \leq t \leq 1$ such that $f(t) \neq 0$.

5. Consider the statement "If x is divisible by 4, then x is divisible by 2". What is its contrapositive ? What is its converse ? What is the contrapositive of the converse? Determine which of the three are true and which are false. Justify with proofs or counterexamples.

6. Do A.9.1, A.9.2.

7. **Fibonacci Numbers** Define a sequence of numbers as follows. $a_1 = 1, a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$ for every $n \geq 3$. Show that $a_n = \frac{\psi^n - \overline{\psi}^n}{\psi - \overline{\psi}}$, where ψ and $\overline{\psi}$ are the roots of the quadratic equation $x^2 = x + 1$, i.e., $\psi = \frac{1+\sqrt{5}}{2}$ and $\overline{\psi} = \frac{1-\sqrt{5}}{2}$. This sequence of numbers is called the *Fibonacci sequence*.

   Hint : First show that $\psi^n = \psi^{n-1} + \psi^{n-2}$ and $\overline{\psi}^n = \overline{\psi}^{n-1} + \overline{\psi}^{n-2}$ for all $n \geq 3$. Then use strong induction.