Practice Midterm II
MAT-165 Mathematics and Computers

Fall 2012

Name _____

- **This test is closed notes, closed book.**
- **There are 6 pages and 5 questions total.**
- **The maximum score in the test is 50 points.**
- **IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM ANOTHER STUDENT'S EXAM. IT IS A VIOLATION OF THE UNIVERSITY'S HONOR CODE TO HAVE ANOTHER PERSON TAKE YOUR EXAM FOR YOU.**

Signature _____

Problem	Score	Max Possible
1		5
2		5
3		10
4		20
5		10
Total		50

1. **(5 pts)** State Buchberger's Criterion.

2. **(5 pts)** Does the S-polynomial $S(f, g)$ for two polynomials f, g depend on the monomial order ? If yes, illustrate with an example. If no, prove it.

3. **(10 pts)** Prove that every minimal Groebner basis of an ideal I has the same number of terms.

4. You can use MAPLE to answer the following questions. Clearly state which MAPLE command was used, the output you got and how that led to your conclusions.

   (a) **(10 pts)** Is the polynomial $p = x^2 + \frac{1}{2}y^2z - z - 1$ in the ideal

   $$I = \langle x^2 + y^2 - 1, x^2 + y^2 + (z-1)^2 - 4 \rangle ?$$

   (b) **(10 pts)** Maximize the value of $x^2 + y^2 + z^2$ subject to the constraint $x^2 + 2y^2 + 3z^2 = 4$.

5. **(10 pts)** Prove or disprove the following statement: Let $F = \{x_1^3 - 1, \ldots, x_n^3 - 1\}$ be a set of polynomials in $\mathbb{C}[x_1, \ldots, x_n]$. F is a Gröbner basis with respect to *any* term order for the ideal $\langle F \rangle$.