Practice Midterm I
MAT-165 Mathematics and Computers

Fall 2012

Name ____

- **This test is closed notes, closed book.**
- **There are 7 pages and 5 questions total.**
- **The maximum score in the test is 70 points.**
- **IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM ANOTHER STUDENT'S EXAM. IT IS A VIOLATION OF THE UNIVERSITY'S HONOR CODE TO HAVE ANOTHER PERSON TAKE YOUR EXAM FOR YOU.**

Signature ____

Problem	Score	Max Possible
1		10
2		30
3		15
4		7
5		8
Total		70

1. (a) **(5 pts)** State Rolle's Theorem for univariate polynomials.

(b) **(5 pts)** State the definition of an ideal of $\mathbb{K}[x_1, \ldots, x_n]$.

2. Using the algorithms we learned in class, answer the following questions.

   (a) **(5 pts)** Determine the number (including multiplicity) of real roots of the polynomial $7x^6 + x^4 - 3x^3 - 5x$. (Dont forget to count 0 as a real root).

   (b) **(10 pts)** Show that there are no common roots to the following set of univariate polynomials: $x^3 + 2x^2 - x - 2, x^3 - 2x^2 - x + 2$ and $x^3 - x^2 - 4x + 5$. You can use MAPLE for this problem, but DO NOT use the *solve*() command. Clearly explain what commands you used, the output given by MAPLE and how that led to your conclusion.

(c) **(15 pts)** Is the polynomial $x^3 - 1$ in the ideal

$$\langle x^3 + 2x^2 - x - 2, x^3 - 2x^2 - x + 2, x^3 - x^2 - 4x + 4\rangle?$$

You can use MAPLE for this problem. Clearly state which functions you used, the output given by MAPLE and how that led to your conclusion.

3. **(15 pts)** Suppose $f_1, \ldots, f_k \in \mathbb{K}[x]$ and $h = GCD(f_1, \ldots, f_k)$. Show that there exist $h_1, \ldots, h_k \in \mathbb{K}[x]$ such that $h = h_1 f_1 + h_2 f_2 + \ldots + h_k f_k$.

4. **(7 pts)** Prove that $\langle 2x^2 + 3y^2 - 11, x^2 - y^2 - 3\rangle = \langle x^2 - 4, y^2 - 1\rangle$.

5. **(8 pts)** Let $S \subseteq \mathbb{R}[x_1, \ldots, x_n]$ be the subset of all polynomials that are divisible by $x_i - 1$ for all $i = 1, \ldots, n$. Show that S is an ideal. (We say that a polynomial $f \in \mathbb{R}[x_1, \ldots, x_n]$ is divisible by $g \in \mathbb{R}[x_1, \ldots, x_n]$ if there exists $q \in \mathbb{R}[x_1, \ldots, x_n]$ such that $f = g \cdot q$.)