Programming Assignment II (due Monday, November 12, 2012)

The following project investigates the geometry of the complex roots of a polynomial $f(x)$ and of its derivative $f'(x)$. You will learn how to use the geometric and graphical capabilities of MAPLE. You will need the *plots* package that you can invoke with the command *with(plots)*. In this package, there are a variety of different commands you may find useful. Try to learn as much as you can about the different options and parameters of the functions that you use for this project.

(a) Write a MAPLE procedure that takes as input a univariate polynomial $f(x)$ and plots the complex roots of a univariate polynomial $f(x)$. You will find the *complexplot()* command useful.

(b) Write a MAPLE procedure that for an arbitrary, not identically constant polynomial $f(x)$, computes and draws the smallest convex polygon containing the zeros of the original polynomial. Such a polygon is called the convex hull. We suggest that to find this polygon you convert complex numbers (the roots of $f(x)$) into vectors, then use the command *convexhull()* included in the package *simplex* to find the desired polygon. You may find the command *polygonplot()* useful.

Let us call the resulting polygon the *Root Polygon* of $f(x)$.

(c) Next consider many polynomials of degree at least 5, whose roots are not all real. Draw in the same picture the root polygons of $f(x)$ and $f'(x), f''(x), f'''(x), \ldots$, etc. until you reach the constant polynomial. Do this for at least 20 different polynomials. Observe what happens. What do you notice? (You do not have to submit any code/output for this part - this is just for your experimentation. Tip: *polygonplot()* can be used to output a list of polygons in the same figure).

(d) Write a MAPLE procedure to automatically generate the pictures above. You should take as input any univariate polynomial $f(x)$, and show all the root polygons for $f(x), f'(x), f''(x), f'''(x), \ldots$, etc. in the same picture. Use colors to make the picture clear. You will have to hand in the MAPLE code that automatically generated your pictures AND write a conjecture based on your empirical observation.