Combinatorial Optimization: AMS 550.666

Amitabh Basu

Department of Applied Mathematics and Statistics, Johns Hopkins U., Fall 2015
Two types of optimization problems

Type I

\( n \) Items:
weights \( w_1, \ldots, w_n \),
values: \( c_1, \ldots, c_n \)
Knapsack with capacity \( W \).
Find the subset with most value that can fit into the knapsack.

Type II

\( n \) Food Items: Each item has price/cost \( p_i \) per unit weight,
nutrition value per unit weight (fat, carbohydrates, vitamins etc.).
Find right combination of food items (weights in pounds) with least cost that meets all nutritional requirements.
Two types of optimization problems

Type I

\( n \) Items:
weights \( w_1, \ldots, w_n \),
values: \( c_1, \ldots, c_n \)
Knapsack with capacity \( W \).
Find the subset with most value that can fit into the knapsack.

Type II

\( n \) Food Items: Each item has price/cost \( p_i \) per unit weight,
nutrition value per unit weight (fat, carbohydrates, vitamins etc.).
Find right combination of food items (weights in pounds) with least cost that meets all nutritional requirements.

1. Brute force approach for Type I does not scale.
2. Classical techniques available for Type II: Calculus, convexity -
   Does not apply to Type I
Two types of optimization problems

Type I
Combinatorial Optimization

\(n\) Items:
weights \(w_1, \ldots, w_n\),
values: \(c_1, \ldots, c_n\)
Knapsack with capacity \(W\).
Find the subset with most value that can fit into the knapsack.

Type II
Continuous Optimization

\(n\) Food Items: Each item has price/cost \(p_i\) per unit weight, nutrition value per unit weight (fat, carbohydrates, vitamins etc.). Find right combination of food items (weights in pounds) with least cost that meets all nutritional requirements.

1. Brute force approach for Type I does not scale.
2. Classical techniques available for Type II: Calculus, convexity - Does not apply to Type I
A Transportation Problem
A Simpler Problem - Transshipment problem

1500
2400
3500
1200
1300
2300
1000
2000
2000
A Scheduling Problem

Job 1
Job 2
Job 3
Job 4

Machine 1
Machine 2
Machine 3
Machine 4
Machine 5
A Problem from Astronomy
A Problem from Astronomy
Use physics to derive an “evaluation” function that evaluates a given partition (Correlation function in astronomy)
A Problem from Astronomy

Use physics to derive an “evaluation” function that evaluates a given partition (Correlation function in astronomy)
A Problem from Astronomy

Use physics to derive an “evaluation” function that evaluates a given partition (Correlation function in astronomy)
A Problem from Astronomy

1000 galaxies: $2^{1000}$ possible partitions
Evaluate a partition in $10^{-20}$ seconds
Will take $\sim 10^{250}$ years!!!!

Use physics to derive an “evaluation” function that evaluates a given partition (Correlation function in astronomy)