1. Let $(X, \mathcal{I})$ be a matroid with $X = \{x_1, \ldots, x_m\}$. Define

$$Y := \{x_i : \text{rk}(\{x_1, \ldots, x_i\}) > \text{rk}(\{x_1, \ldots, x_{i-1}\})\}.$$ 

Prove that $Y \in \mathcal{I}$.

2. Show that a partition matroid is also a graphic matroid (you can use multiple edges to define your graph), and a linear matroid.

3. Show that every graphic matroid is a linear matroid.

4. Let $G = (V, E)$ be a graph. Let $X = E$ and $\mathcal{I} = \{J \subseteq E : J \text{ contains at most one cycle}\}$. Show that $(X, \mathcal{I})$ is a matroid.