AMS 550.472/672: Graph Theory
Homework Problems - Week VII

Problems to be handed in on Wednesday, March 23 in class: 4.1.28 from textbook, Problem 5 below. A wording of Problem 4.1.28 is given below.

> Let $G = (V, E)$ be a simple undirected graph. Let $A, B \subseteq E$ be two different edge cuts of G. Show that the set of edges $A \Delta B$ is an edge cut. Recall that $A \Delta B := (A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$.

1. Give an example of a graph G whose minimum degree is at least 3, and yet $\kappa(G) = 1$. For every natural number $k \geq 2$, give an example of a graph G whose minimum degree is at least k, and yet $\kappa(G) = 1$.

2. Do Problems 4.1.1 a), b). ("connectivity" means $\kappa(G)$ in the textbook)

3. Do Problems 4.1.19. ($\delta(G)$ denotes the minimum degree in the textbook).

4. Do Problems 4.1.14, 4.1.26, 4.1.28. (k-connected means k-vertex connected in the textbook).

5. Let X be a finite set. Let $A_1, A_2, \ldots, A_n, B_1, B_2, \ldots, B_n$ be subsets of X. A subset $Y \subseteq X$ is called a *common system of distinct representatives* if there exist two bijections $f, g : \{1, \ldots, n\} \to Y$ such that $f(i) \in A_i$ for $i = 1, \ldots, n$ ($f(i)$ is called the representative of A_i), and $g(j) \in B_j$ for $j = 1, \ldots, n$ ($g(j)$ s called the representative of B_j). (Note that a particular $y \in Y$ may represent A_i and B_j with $i \neq j$.

   Show that a collection of subsets $A_1, A_2, \ldots, A_n, B_1, B_2, \ldots, B_n$ has a common system of representative if and only if

   $$|(\bigcup_{i \in I} A_i) \cap (\bigcup_{j \in J}) B_j| \geq |I| + |J| - n$$

   for each pair of subsets $I, J \subseteq \{1, \ldots, n\}$. [Hint: Set up a particular graph that models these set interactions. The graph will have two special vertices. Use Menger's Theorem on the two vertices.]