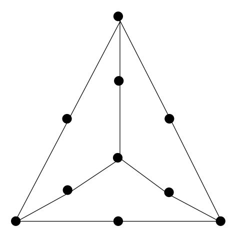
AMS 550.472/672: Graph Theory
Homework Problems - Week VI

1. Do Problems 3.2.3, 3.2.4, 3.2.11.

2. Check the Tutte-Berge formula for an odd cycle, i.e., show that for an odd cycle C,

$$\nu(C) = \min_{\emptyset \subseteq A \subseteq V(C)} \frac{|V| - oc(V(C) \setminus A) + |A|}{2}.$$

3. Determine whether the graph below has a perfect matching. Justify your answer. Find a maximum matching in the graph and give a short argument to show that it is indeed maximum.



4. Let $G = (V(G), E(G))$ be a general graph and $k \leq |V(G)|/2$ be a given positive integer. Construct a graph G' such that G' has a perfect matching if and only if G has a matching of size k. [Hint: Add some vertices and edges to G]

5. Show that Tutte's perfect matching condition implies the Tutte-Berge formula. (Thus, they are equivalent) [Hint: Use the previous exercise]

6. Let $G = (V(G), E(G))$ be a simple graph and let $T \subseteq V(G)$ be a subset of vertices. Show that there is a matching M in G such that all vertices in T are covered by some edge in M if and only if for every $\emptyset \subseteq W \subseteq V(G)$, the number of odd sized components in $G \setminus W$ that are fully contained in T is at most $|W|$. [Hint: Add some vertices and/or edges to G]

7. Construct a 3-regular simple graph with a cut edge.

8. Show that a tree T has a perfect matching if and only if for every vertex v, $T \setminus \{v\}$ has exactly one odd component.