



Syllabus
Applied Mathematics and Statistics 553.465/553.665
Introduction to Convexity
Fall, 2018

Description The main goal of this course is to introduce students to the structural and algorithmic aspects of convex geometry and analysis. The course will have roughly 3 modules: 1. Structure and properties of convex sets, 2. structure and properties of convex functions, 3. Introduction to convex optimization with an emphasis on algorithms for linear optimization/programming (LPs).

Prerequisites

No formal prerequisites. Familiarity with the basic notions of the following will be helpful.

Linear Algebra (EN.550.291 or AS.110.201 or AS.110.212)

Real Analysis (EN.550.405)

Instructor

Amitabh Basu, basu.amitabh@jhu.edu, <http://www.ams.jhu.edu/~abasu9/>

Office: Whitehead 202A, 410-516-4883

Office hours: Wednesdays 5:00 - 7:00 pm or by appointment. Email me to schedule an appointment outside the listed hours. Office hours will be in the instructor's office: Whitehead 202A.

Teaching Assistant

Ning Liu, nliu15@jhu.edu

Office hours: Ning's office hours will be on Thursdays, 5:00 – 6:30pm, in Whitehead 212.

Meetings

Lectures: Tuesday, Thursday, 3:00-4:15 pm, Bloomberg 274.

Discussions: Friday, 12:00 - 12:50 pm, Bloomberg 178.

Textbook

I will post my own lecture notes on the course webpage: http://www.ams.jhu.edu/~abasu9/AMS-553_465_F17.html

Other useful textbooks (but not required):

Convex Analysis and Minimization Algorithms, vol. I by Hirriat-Urruty and Lemarechal; Springer-Verlag, ISBN 0-387-56852-2.

A Course in Convexity by Alexander Barvinok; AMS Graduate Studies in Mathematics, ISBN 0-8218-2968-8.

Convex Analysis by R. T. Rockafeller; Princeton Landmarks in Mathematics, ISBN 0-691-01586-4.

Convex and Discrete Geometry by Peter Gruber; Springer, ISBN 978-3-540-71132-2. Also available for free online from within JHU.

Online Resources

Course webpage: http://www.ams.jhu.edu/~abasu9/AMS-553_465_F18.html

Blackboard will be used for posting grades.

Course Objectives

The main goal of this course is to introduce students to the structural and algorithmic aspects of convex geometry and analysis. The course will have roughly 3 modules: 1. Structure and properties of convex sets, 2. structure and properties of convex functions, 3. Introduction to convex optimization with an emphasis on algorithms for linear optimization/programming (LPs).

Course Topics

PART I: Structure and Properties of Convex Sets

- (1) Convex sets: definition, examples, basic results, projections, extreme points, cones, convex cones, recession cones, normal cone, tangential cone, dual cone, polar cones.
- (2) Affine combinations and dimensions of convex sets.
- (3) Separation type results:
 - (i) Separating hyperplane result.
 - (ii) Farkas.
 - (iii) Polarity related to convex sets.
- (4) Basic Polyhedral Theory
 - (i) Definitions: polyhedra, faces, vertex=extreme point
 - (ii) Minkowski-Weyl theorem.
- (5) Combinatorial Theorems: Helly-Caratheodory-Radon.

PART II: Structure and Properties of Convex Functions

- (1) Continuity, differentiability, basic results, second-order (Hessians), first-order (gradients) and zeroth order characterizations via the epigraph.
- (2) Conjugate functions.
- (3) Sublinearity; gauge and support functions
- (4) Subdifferential calculus

PART III: Convexity in optimization

- (1) Linear Programming :
 - Duality theory
 - Algorithms for LP – simplex, interior point methods, ellipsoid method.
 - Equivalence of optimization and separation
- (2) General Convex optimization techniques:
 - Subgradient methods, interior-point methods, cutting plane methods.
 - conic optimization: semidefinite programming, second-order cone programming
 - general Lagrangean duality

A detailed lecture-by-lecture schedule or topics can be found here: http://www.ams.jhu.edu/~abasu9/AMS-553_465/schedule-2018.html.

Course Expectations & Grading

There will one take home Midterm and one take home Final exam.

In addition, there will be regular (approx. weekly) homework assignments. You will be asked to hand in some of the HW problems which will be graded (approximately every two weeks). Seriously attempting ALL the homework problems is imperative for your success in the class, and they will give an indication

of the kind of problems on the tests. The HW problems will appear here on the course webpage: http://www.ams.jhu.edu/~abasu9/AMS-553_465_F18.html.

Key Dates

The Midterm will be take home. It will be posted on the course webpage by Friday, October 19, 2018 (noon). It will be due back Thursday, October 25, 2018 at the beginning of class.

Final Exam will be take home. It will be posted on the course webpage at the beginning of the Reading Period and will be due back by 5 pm on Wednesday, December 12, 2018.

Assignments & Readings

See the course webpage: http://www.ams.jhu.edu/~abasu9/AMS-553_465_F18.html for HW assignments.

Ethics

The strength of the university depends on academic and personal integrity. In this course, you must be honest and truthful. Ethical violations include cheating on exams, plagiarism, reuse of assignments, improper use of the Internet and electronic devices, unauthorized collaboration, alteration of graded assignments, forgery and falsification, lying, facilitating academic dishonesty, and unfair competition.

In addition, the specific ethics guidelines for this course are:

- (1) You may discuss HW problems with your fellow classmates. However, you have to write your own final solutions without looking at anyone else's solutions.
- (2) MIDTERM RULES:
 - You are not allowed to discuss any problem with another human being (this includes your classmates, of course), except Dr. Basu.
 - You can use a computer only as a word processor; in particular, you cannot consult the internet in regards to this midterm. You CAN use any other resource like the textbook, your notes, HW solutions, books from the library.
 - You CAN cite any result we have mentioned in class or from the HWs without proof. If you cite a result (e.g., from a book) that was NOT mentioned in class, you should include a complete proof of this fact.
- (3) FINAL RULES:
 - You are not allowed to discuss any problem with another human being (this includes your classmates, of course), except Dr. Basu or the TA.
 - You can use a computer only as a word processor; in particular, you are not allowed to access the internet during your final. You CAN use any other resource like the textbook, your notes, HW solutions, books from the library.
 - You CAN cite any result we have mentioned in class or from the HWs without proof. If you cite a result (e.g., from a book) that was NOT mentioned in class, you should include a complete proof of this fact.

Report any violations you witness to the instructor.

You can find more information about university misconduct policies on the web at these sites:

- Undergraduates: e-catalog.jhu.edu/undergrad-students/student-life-policies/
- Graduate students: e-catalog.jhu.edu/grad-students/graduate-specific-policies/

Students with Disabilities

Any student with a disability who may need accommodations in this class must obtain an accommodation letter from Student Disability Services, 385 Garland, (410) 516-4720, studentdisabilityservices@jhu.edu.

ABET Outcomes

- Ability to apply mathematics, science and engineering principles.
- Ability to identify, formulate and solve engineering problems.
- Understanding of professional and ethical responsibility.
- Ability to communicate effectively.