Homework No.7, 550.386, Due Thursday, April 24, 2008.

1. Atkinson, Problem 6.14.

2. Prove Atkinson's equation (6.4.17). More precisely, take $h = t/n$ and show that

$$r_0^n = \exp[\lambda t + O((\lambda t)^3/n^2)].$$

Hint: Consider $\ln(r_0^n)$ and use $\text{arcsinh}\,(x) = \ln(x + \sqrt{1+x^2}) = x - x^3/6 + O(x^5)$.

3. Write a code `trapezoid2.m` that implements the trapezoidal method, but which uses the iteration scheme in Atkinson eq.(6.5.3) rather than the Newton method in order to solve for the fixed-point at each time-step. Apply both your code and the original code `trapezoid.m`, using the Newton method, to the initial-value problem

$$\begin{cases} dy/dt = -y, \quad 0 < t < 1 \\ y(0) = 1 \end{cases}$$

using $N_s = 25$ steps. Verify that the two codes give the same result, to the requested tolerance $TOL = 10^{-15}$ in the root-finding iteration. Also, report the average number of iterations per time-step for the two root-finding algorithms.

4. Atkinson, Problem 6.16 (c),(d). However, for each of these two initial-value problems, use all three of the methods midpoint, PECE, and trapezoidal, for numbers of steps $N_s = 10, 100, 1000$. For each of these cases (eighteen in all!) plot the numerical solution and its exact error. Compare and discuss the performance of the three methods. Do all three appear to converge? Which method performs better at finite time-step h, under what circumstances, and why?

5. This problem concerns the 4th-order Simpson rule multistep method discussed by Atkinson in Section 6.7:

$$\mathbf{y}_{n+1} = \mathbf{y}_{n-1} + \frac{h}{3}[\mathbf{f}(t_{n-1}, \mathbf{y}_{n-1}) + 4\mathbf{f}(t_n, \mathbf{y}_n) + \mathbf{f}(t_{n+1}, \mathbf{y}_{n+1})].$$

(a) Find the roots $r_0(h\lambda), r_1(h\lambda)$ of the characteristic equation for this method.

(b) Use the results in part (a) to show that this method is not relatively stable.