Homework No.1, 550.386, Due February 7, 2008.

1. Atkinson, Problem 1.4: Assuming $g \in C[a,b]$, show for $h = b - a$ that

$$\int_a^b (x-a)^2(b-x)^2 g(x)dx = \frac{h^5}{30} g(\xi) \quad \text{for some} \quad x \in [a,b].$$

2. Atkinson, Problem 1.5(d).

3. Convert the following numbers to their decimal equivalent:

(a) $(110011.0011)_2$ (b) $(aa88.ff)_{16}$
(c) $(0.ccc\cdots)_{16}$ (d) $(\underbrace{1\cdots1}_{n})_2$

4. Show that to convert between binary and hexadecimal representations, each hecadecimal digit b is replaced, in order, by the four binary digits a, a', a'', a''' that satisfy

$$b = 2^3 a + 2^2 a' + 2a'' + a'''.$$

Use this fact to construct a simple algorithm to convert a hexadecimal representation to a binary representation for both integers and fractions, as in Atkinson, Problems 1.11 and 1.12.

5. In IEEE Standard Floating-Point representation for numbers, a 12-bit binary number is used first to encode the sign σ and exponent E, followed by a 52-bit binary to encode the fraction F. This can also be represented by a 3-digit hexadecimal number, followed by a 13-digit hexadecimal number, as in `MATLAB`. Consider the following IEEE Standard Floating-Point number in hexadecimal form:

```
x=c001e3779b97f4a8
```

(a) Use the first 3 hexadecimal digits to determine the sign σ and exponent E of `x` in decimal format.

(b) Convert the remaining 13 hexadecimal digits to a decimal representation of the fraction F for `x`.

(c) Calculate the decimal representation of the double-precision number `x`.

6. Atkinson, Problem 1.16.

7. For the following numbers x_A and x_T, how many significant digits are there in x_A with respect to x_T?

$$\text{(a) } x_A = 0.7854, \quad x_T = \pi/4$$
$$\text{(b) } x_A = 3.7612, \quad x_T = \ln(43)$$

8. Atkinson, Problem 1.23.

9. Give exact ways to avoid loss-of-signficance errors in the following computations:
(a) $\frac{e^x - e^{-x}}{2x}$ for $x \doteq 0$ (*Hint:* Use a Taylor approximation.)
(b) $\cos(x) - \cos(y)$ for $x \doteq y$ (*Hint:* Use a trigonometric identity.)

10. Atkinson, Problem 1.30(a).