

Linear Algebra and Differential Equations (550.291)
Homework 1 (Due Thursday, September 22, 2005)

General Directions: You must show all work and document any assumptions to receive full credit.

Basic Definitions and Terminology

1. State whether the given differential equations are linear or nonlinear. Give the order of each equation.

(a)

$$(1 - x)y'' - 4xy' + 5y = \cos x$$

(b)

$$x \frac{d^3y}{dx^3} - 2 \left(\frac{dy}{dx} \right)^4 + y = 0$$

(c)

$$\frac{dy}{dx} = \sqrt{1 + \left(\frac{d^2y}{dx^2} \right)^2}$$

2. Verify the indicated function is a solution of the given differential equation.

(a)

$$2y' + y = 0; \quad y = e^{-x/2}$$

(b)

$$y' + y = \sin x; \quad y = 0.5 \sin x - 0.5 \cos x + 10e^{-x}$$

Separation of Variables

3. Find a general solution to the given differential equation by separation of variables.

(a)

$$\frac{dx}{dy} = \frac{1 + 2y^2}{y \sin x}$$

- (b) After obtaining a general solution, apply the initial condition to obtain a particular solution.

$$\frac{dy}{dx} = \frac{y^2 - 1}{x^2 - 1}; \quad y(2) = 2$$

- (c) After obtaining a general solution, apply each initial condition to obtain a particular solution. Plot the three solution functions on the same graph.

$$\frac{dy}{dx} - y^2 = -9; \quad (i)(0, 0); \quad (ii)(0, 3); \quad (iii)(1/3, 1)$$

First Order Linear ODEs

4. Find a general solution to the given differential equation.

(a)

$$x^2y' + xy = 1$$

(b)

$$(x + 4y^2)dy + 2ydx = 0$$

- (c) After obtaining a general solution, apply the initial condition to obtain a particular solution.

$$(x + 1)\frac{dy}{dx} + y = \ln x; \quad y(1) = 10$$