Solution HW-9 (550.291)

1) 7.2.21

$$W(t) = \begin{vmatrix} 3e^{-2t} & e^{t} & e^{3t} \\ -2e^{-2t} & -e^{t} & -e^{3t} \\ 2e^{-2t} & e^{t} & 0 \end{vmatrix} = e^{2t} \neq 0.$$

$$x(t) = c_1 x_1 + c_2 x_2 + c_3 x_3$$

$$= \begin{bmatrix} 3c_1 e^{-2t} + c_2 e^{t} + c_3 e^{3t} \\ -2c_1 e^{-2t} - c_2 e^{t} - c_3 e^{3t} \\ 2c_1 e^{-2t} + c_2 e^{t} \end{bmatrix}.$$

2) <u>7.2.29</u>

$$x_1(t) = 3c_1 e^{-2t} + c_2 e^{t} + c_3 e^{3t}.$$

$$x_2(t) = -2c_1 e^{-2t} - c_2 e^{t} - c_3 e^{3t}.$$

$$x_3(t) = 2c_1 e^{-2t} + c_2 e^{t}.$$

$$3c_1 + c_2 + c_3 = 1, \quad -2c_1 - c_2 - c_3 = 2, \quad 2c_1 + c_2 = 3$$

$$x_1(t) = 9e^{-2t} - 3e^{t} - 5e^{3t}.$$

$$x_2(t) = -6e^{-2t} + 3e^{t} + 5e^{3t}$$

$$x_3(t) = 6e^{-2t} - 3e^{t}.$$

3) 7.3.9

characteristic equation $\lambda^2 + 16 = 0$

Eigenvalue $\lambda = 4i$

Eigenvector equation. $\begin{bmatrix} 2-4i & -5 \\ 4 & -2-4i \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Eigenvector $v = [5 \quad 2-4i]^T$

The real and imaginary parts of.

$$x(t) = v e^{4it} = \begin{bmatrix} 5\cos 4t + 5i\sin 4t \\ (2\cos 4t + 4\sin 4t) + i(2\sin 4t - 4\cos 4t) \end{bmatrix}$$

yield the general solution.

$x_1(t) = 5c_1 \cos 4t + 5c_2 \sin 4t$

$x_2(t) = c_1(2\cos 4t + 4\sin 4t) + c_2(2\sin 4t - 4\cos 4t)$

The initial conditions $x_1(0) = 2$ and $x_2(0) = 3$ give $c_1 = 2/5$ and $c_2 = -11/20$, so the desired particular solution is

$$x_1(t) = 2\cos 4t - \frac{11}{4}\sin 4t$$

$$x_2(t) = 3\cos 4t + \frac{1}{2}\sin 4t$$

4) 7.3.21

characteristic equation $\rightarrow -\lambda^3 + \lambda = 0.$

Eigenvalues $\lambda_1 = 0, \lambda_2 = 1, \lambda_3 = -1$

Eigenvector equations.

$$\begin{bmatrix} 5 & 0 & -6 \\ 2 & -1 & -2 \\ 4 & -2 & -4 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & -6 \\ 2 & -2 & -2 \\ 4 & -2 & -5 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 0 & -6 \\ 2 & 0 & -2 \\ 4 & -2 & -3 \end{bmatrix} \begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Eigenvectors $v_1 = [6 \quad 2 \quad 5]^T \quad v_2 = [3 \quad 1 \quad 2]^T$

$v_3 = [2 \quad 1 \quad 2]^T$

$x_1(t) = 6c_1 + 3c_2 e^t + 2c_3 e^{-t}.$

$x_2(t) = 2c_1 + c_2 e^t + c_3 e^{-t}.$

$x_3(t) = 5c_1 + 2c_2 e^t + 2c_3 e^{-t}.$

5) 7-3-29

The coefficient matrix.

$$A = \begin{bmatrix} -0.2 & 0.4 \\ 0.2 & -0.4 \end{bmatrix}$$

has eigenvalues $\lambda_1 = 0$ and $\lambda_2 = -0.6$. with eigenvectors $v_1 = [2 \quad 1]^T$ and $v_2 = [1 \quad -1]^T$ that yield general solution.

$$x_1(t) = 2c_1 + c_2 e^{-0.6t}.$$

$$x_2(t) = c_1 - c_2 e^{-0.6t}.$$

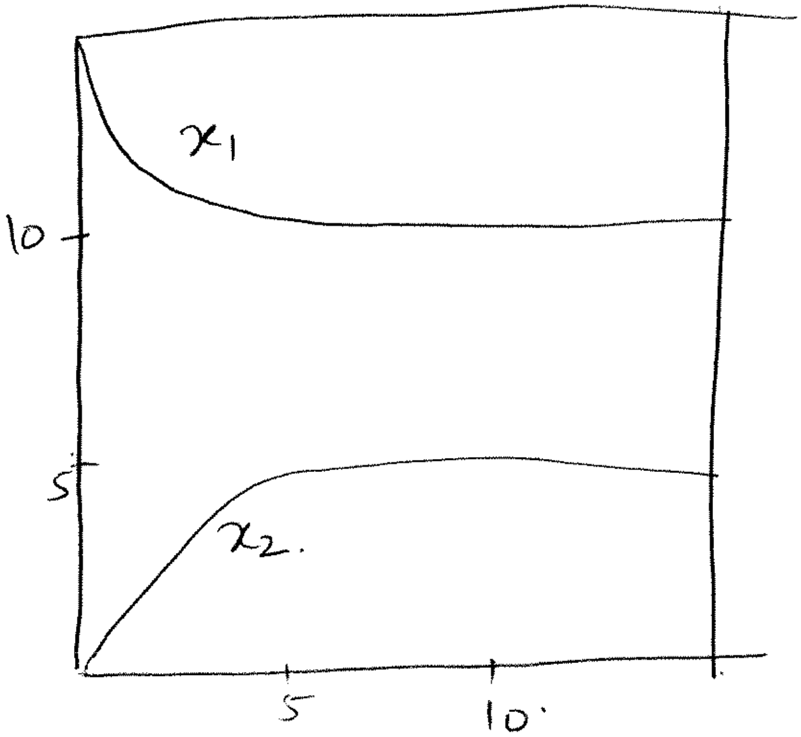
The initial conditions $x_1(0) = 15$, $x_2(0) = 0$ give $c_1 = c_2 = 5$.

So we get

$$x_1(t) = 10 + 5e^{-0.6t}.$$

$$x_2(t) = 5 - 5e^{-0.6t}.$$

The graph of $x_1(t)$ and $x_2(t)$ is on the next page.



6) 7.5.15

$u_1 = [3 \quad -1 \quad 0]^T \qquad u_2 = [0 \quad 0 \quad 1]^T$

$v_1 = [-3 \quad 1 \quad 1]^T \qquad v_2 = [1 \quad 0 \quad 0]^T$

$x_1(t) = e^t(3c_1 + c_3 - 3c_3 t)$

$x_2(t) = e^t(-c_1 + c_3 t)$

$x_3(t) = e^t(c_2 + c_3 t)$

7) 7.5.37

$v_1 = [1 \quad -1 \quad 2]^T$, $v_2 = [4 \quad 0 \quad 9]^T$,

$v_3 = [0 \quad 2 \quad 1]^T$

$$Q = [v_1 \quad v_2 \quad v_3] = \begin{bmatrix} 1 & 4 & 0 \\ -1 & 0 & 2 \\ 2 & 9 & 1 \end{bmatrix}$$

$$J = Q^{-1} A Q$$

$$= \frac{1}{2} \begin{bmatrix} -18 & -4 & 8 \\ 5 & 1 & -2 \\ -9 & -1 & 4 \end{bmatrix} \begin{bmatrix} 39 & 8 & -16 \\ -36 & -5 & 16 \\ 72 & 16 & -29 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ -1 & 0 & 2 \\ 2 & 9 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$