Solution HW 7

LADE

1) (a) 5.1.17.

If $y = \frac{c}{x}$, then $y' + y^2 = -\frac{c}{x^2} + \frac{c^2}{x^2} = \frac{c(c-1)}{x^2} \neq 0$.

unless either $c = 0$ or $c = 1$

(b) 5.1.18

If $y = cx^3$ then $yy'' = cx^3 \, 6cx$

$= 6c^2x^4 \neq 6x^4$ unless $c^2 = 1$

(c) 5.1.19

If $y = 1+\sqrt{x}$ then $yy'' + (y')^2$

$= (1+\sqrt{x})(-x^{-3/2}/4) + (x^{-1/2}/2)^2$

$= -x^{-3/2}/4 \neq 0$

2)(a) Let $L[y] = y'' + py' + qy$. Then $L[y_c] = 0$

and $L[y_p] = f$, so

$L[y_c + y_p] = L[y_c] + L[y_p] = 0 + f = f$.

(b) If $y(x) = 1 + c_1 \cos x + c_2 \sin x$, then ~~$y'(x)$~~

$y'(x) = -c_1 \sin x + c_2 \cos x$, so initial conditions $y(0) = y'(0) = -1$ yield

$c_1 = -2$, $c_2 = -1$. Hence

$y = 1 - 2\cos x - \sin x$.

3) 5.1.39.

$4r^2+4r+1=0$

$r = -1/2, -1/2.$

$y(x) = (c_1+c_2x)e^{-x/2}.$

4) 5.1.56.

The substitution $v = \ln x$ yields.

$d^2y/dv^2 - 4\frac{dy}{dv} + 4y = 0$

whose characteristic equation

$r^2 - 4r+4 = 0$ has roots $r_1, r_2 = 2.$

Because $e^v = x$, the corresponding general solution is

$y = c_1e^{2v} + c_2ve^{2v} = x^2(c_1+c_2\ln x)$

5) 5.2.9

$W = e^x(\cos^2x + \sin^2x) = e^x \neq 0$

6) 5.3.36

The fact that $y = e^{-x}\sin x$ is one tells us that $(r+1)^2+1 = r^2+2r+2$ is a factor of characteristic polynomial.

$9r^3 + 11r^2 + 4r - 14$

Then long division yields linear factor $9r-7$. Hence general solution is

$$y(x) = c_1 e^{7x/9} + e^{-x}(c_2 \cos x + c_3 \sin x)$$