

Solution to Assignment 3

SSo. 291

1) 3.2.15

Begin by interchanging 1 and 2 of A. Then subtract 3 times row 1 from row 2, and five times row 1 from row 3

$$E = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The system has no solution.

2) 3.2.20

Begin by interchanging rows 1 and 2 of A. Then subtract twice row 1 from row 2, and five times row 1 from row 3.

$$E = \begin{bmatrix} 1 & 3 & 2 & -7 & 3 & 9 \\ 0 & 1 & 3 & -7 & 2 & 7 \\ 0 & 0 & 1 & -2 & 0 & 2 \end{bmatrix}$$

$$x_1 = 2 + 3t$$

$$x_2 = 1 + s - 2t$$

$$x_3 = 2 + 2s$$

$$x_4 = s$$

$$x_5 = t$$

3) 3.3.19

SWAP R_1, R_3

$$\begin{bmatrix} 2 & 7 & -10 & -19 & 13 \\ 1 & 3 & -4 & -8 & 6 \\ 1 & 0 & 2 & 1 & 3 \end{bmatrix} \xrightarrow{\text{SWAP } R_1, R_3} \begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 1 & 3 & -4 & -8 & 6 \\ 2 & 7 & -10 & -19 & 13 \end{bmatrix}$$

$$\xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 0 & 3 & -6 & -9 & 3 \\ 2 & 7 & -10 & -19 & 13 \end{bmatrix} \xrightarrow{R_3 - 2R_1} \begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 0 & 3 & -6 & -9 & 3 \\ 0 & 7 & -14 & -21 & 7 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{3}R_2} \begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 0 & 1 & -2 & -3 & 1 \\ 0 & 7 & -14 & -21 & 7 \end{bmatrix} \xrightarrow{R_3 - 7R_2} \begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 0 & 1 & -2 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4) 3.3.32

If $ad - bc \neq 0$, then not both a and b can be zero. If, for instance, $a \neq 0$, then

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \xrightarrow{(\frac{1}{a})R_1} \begin{bmatrix} 1 & b/a \\ c & d \end{bmatrix} \xrightarrow{R_2 - cR_1} \begin{bmatrix} 1 & b/a \\ 0 & d - bc/a \end{bmatrix}$$

$$\xrightarrow{aR_2} \begin{bmatrix} 1 & b/a \\ 0 & ad - bc \end{bmatrix} \xrightarrow{\frac{1}{(ad-bc)}R_2} \begin{bmatrix} 1 & b/a \\ 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_1 - (b/a)R_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

5) 3.5.8

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 10 & -15 \\ -5 & 8 \end{bmatrix}$$

$$x = \frac{1}{5} \begin{bmatrix} 10 & -15 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 25 \\ -11 \end{bmatrix}$$

6) 3.5.16

$$\begin{bmatrix} 1 & -3 & -3 & 1 & 0 & 0 \\ -1 & 1 & 2 & 0 & 1 & 0 \\ 2 & -3 & -3 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2+R_1} \begin{bmatrix} 1 & -3 & -3 & 1 & 0 & 0 \\ 0 & -2 & -1 & 1 & 1 & 0 \\ 2 & -3 & -3 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3-2R_1} \begin{bmatrix} 1 & -3 & -3 & 1 & 0 & 0 \\ 0 & -2 & -1 & 1 & 1 & 0 \\ 0 & 3 & 3 & -2 & 0 & 1 \end{bmatrix} \xrightarrow{R_2+R_3} \begin{bmatrix} 1 & -3 & -3 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 1 \\ 0 & 3 & 3 & -2 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3-3R_2} \begin{bmatrix} 1 & -3 & -3 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 1 \\ 0 & 0 & -3 & 1 & -3 & -2 \end{bmatrix} \xrightarrow{-\frac{1}{3}R_3} \begin{bmatrix} 1 & -3 & -3 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 1 \\ 0 & 0 & 1 & -\frac{1}{3} & 1 & \frac{2}{3} \end{bmatrix}$$

$$\xrightarrow{R_1+3R_2} \dots \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -\frac{1}{3} & -1 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{3} & 1 & \frac{2}{3} \end{bmatrix} \text{ so } A^{-1} = \frac{1}{3} \begin{bmatrix} -3 & 0 & 3 \\ -1 & -3 & -1 \\ -1 & 3 & 2 \end{bmatrix}$$

7) 3.5.37

Direct multiplication shows that
 $AA^{-1} = A^{-1}A = I$.