Linear Algebra and Differential Equations (550.291)
Homework 2 (Due Tuesday, February 12, 2008)

General Directions: You must show all work and document any assumptions to receive full credit. All problems are to be done by hand unless otherwise stated.

Please make sure to check the schedule (available online) for the assigned readings for this week. (Homework may cover material presented in lecture up to the lecture 2 days before the assignment is due.)

1. A first-order equation of the form
$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$
is called a **Bernoulli equation**. (See pages 62-53 for examples.) As described in the text, that for any $n \neq 0, n \neq 1$, the substitution $v = y^{1-n}$ converts a Bernoulli equation into a linear equation. Show that the equation below is a Bernoulli equation and then solve it.
$$3y^2y' - 2y^3 - x - 1 = 0, \quad y(0) = 2$$

2. Show that the following equation is exact and then solve.
$$2xydx + (x^2 + 1)dy = 0$$

3. It is sometimes possible to transform a nonexact differential equation of the form
$$\tilde{M}(x, y)dx + \tilde{N}(x, y)dy = 0$$
into an exact equation by multiplying it by an integrating factor $\rho(x, y)$. The resulting equation,
$$M(x, y)dx + N(x, y)dy = 0$$
where $M(x, y) = \rho(x, y)\tilde{M}(x, y)$ and $N(x, y) = \rho(x, y)\tilde{N}(x, y)$, is exact. For the ODE below, show that (a) the original equation is not exact; (b) the given function $\rho(x, y)$ is an integrating factor that makes the resulting equation exact; (c) solve the resulting equation; and (d) verify that the solution from (c) solves the original ODE.
$$(3y^2 + 5x^2y)dx + (3xy + 2x^3)dy = 0, \quad \rho(x, y) = x^2y$$

4. Edwards & Penney: Problem 3.4.23

5. Edwards & Penney: Problem 3.4.27

6. Edwards & Penney: Problem 3.4.31

7. Edwards & Penney: Problem 3.4.42(a)

8. Edwards & Penney: Problem 3.1.28

MATLAB Practice
You do not have to turn in this portion of the assignment. However, you are expected to know how to use MATLAB to perform the operations used below.

Open a workspace in MATLAB by selecting the program from the Start Menu.

Part 1: Vector Operations.

(a) Enter the row vector $v = [3\ 2\ \ -7]$ by typing `v = [3,2,-7]`

(b) Convert v to a column vector by typing `v = v'` .

(c) Compute $2v$ by typing `2*v`.

(d) Enter the **column vector** $w = \begin{bmatrix} -4 \\ 0 \\ 6 \end{bmatrix}$.

(e) Compute $v + w$ by typing `v + w` .

(f) Compute the vector formed by cubing each element of w (type `w.^3`). The "." before the operator causes each element of the vector to undergo the operation (see what happens if you type `w^2`).

(g) Compute the vector formed by inverting each element of v (type `1./v`).

(h) Compute the product $v^T w$ (type `v'*w`).

(i) Compute the vector u where $[u_j] = [v_j w_j]$ (type `u = v.*w`)

(j) Sum all the elements in v (type `sum(v)`).

(k) Create a zero column vector $x \in \mathbb{R}^4$ (type `x = zeros(4,1)`).

(l) Create a column vector $x \in \mathbb{R}^4$ of all ones (type `x = ones(4,1)`).

(m) Assign the values 0, 0.1, 0.2, ..., 1 to the vector x (type `x = 0:0.1:1`).

(n) Make x a column vector.

Part 2: Matrix Operations.

(a) Enter the matrix $A = \begin{bmatrix} 2 & 3 & 5 \\ 5 & 1 & 8 \\ 12 & 5 & 21 \end{bmatrix}$ (type `A=[2, 3, 5; 5, 1, 8; 12, 5, 21]`).

(b) Show the first row of A by typing `A(1,:)` .

(c) Let's find the transpose of A. Let $B = A^T$ (type `B = A'`).

(d) Show the second column of B by typing `B(:,2)` .

(e) Enter the matrix $C = \begin{bmatrix} 2 & 5 & 1 \\ 3 & 1 & 5 \\ 5 & 8 & 5 \end{bmatrix}$

(f) Find the solution y to the system $Cy = v$ by typing `y = C\v`

(g) Compute the product AB (type $A * B$).

(h) Create a 3×3 identity matrix by typing `eye(3)`