LADE (550.251) Quiz 5

NAME Solutions

Show that the vectors {(1,0,1), (0,1,1), (0,-1,0)} form a basis for $\mathbf{R}^3$ and then express (-1,2,3) as a linear combination of these vectors.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & -1 & b \\ 1 & 1 & 0 & c \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & -1 & b \\ 0 & 1 & 0 & c-a \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & -1 & b \\ 0 & 0 & 1 & c-a-b \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & c-a \\ 0 & 0 & 1 & c-a-b \end{array}\right]$$

Let $(a,b,c) = (0,0,0)$ then the only soln to

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ is $c_1 = a = 0$, $c_2 = c-a = 0$, $c_3 = c-a-b = 0$

the 3-vectors are linearly indep. Since $\dim(\mathbb{R}^3) = 3$ these vectors must form a basis for $\mathbb{R}^3$.

To write $(a,b,c) = (-1,2,3)$ as a linear combo of the 3 vectors set $c_1 = a = -1$ $c_2 = c-a = 3-(-1) = 4$ $c_3 = c-a-b = 2$

So $$-1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + 4 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$ as desired.