# 550.251 Practice Problems for Exam II

Problem 1

A nightwatchman must make his rounds to three areas of a plant every hour on the hour. When he reaches an area, he inspects it, which takes 5 minutes, then punches a timeclock to verify his inspection. After visiting all three areas, he returns to the lobby and watches television. The times between locations are symmetric. They are given in minutes in the table presented below.

| | *Area 1* | *Area 2* | *Area 3* |
|---|---|---|---|
| *Lobby* | 10 | 4 | 8 |
| *Area 1* | | 7 | 9 |
| *Area 2* | | | 6 |

(a) In which order should he inspect the areas if his objective is to maximize his television time? (Use exhaustive enumeration)

(b) His favorite one-hour television show, *Nightwatchman: Man of Mystery*, starts at 9:00pm. How much of this show will he get to see?

Problem 2

Sue has to solve the following integer linear program (ILP):

$$\max x_1 + 5x_2$$

subject to

$$\begin{aligned} -4x_1 + 2x_2 &\leq 6 \\ 3x_1 + 2x_2 &\leq 18 \end{aligned}$$

$$x_1, x_2 \geq 0, \quad \text{integer}$$

She decided to first solve the LP relaxation and then round the LP's solution (if necessary)

a) What are the differences between the formulation of the LP relaxation and that of the original ILP?

b) The optimum value of the objective function for the original ILP will be (circle one)

$$\geq \qquad\qquad \leq$$

the optimum value of the objective function for the LP relaxation. Explain your answer.

c) The optimum solution to the LP relaxation is $x^* = (x_1^*, x_2^*) = (1.71, 6.43)$. To obtain an integer solution, Sue decided to round **UP** (i.e., round $x_1$ and $x_2$ to the smallest integer greater than or equal to the numbers given.) Call this rounded solution $\hat{x}$.

What is $\hat{x}$?

d) Might $\hat{x}$ be optimal for the ILP? Explain your answer.

Problem 3

Oilco must determine how many barrels of oil to extract during each of the next 2 years. If Oilco extracts $x_1$ million barrels during year 1, each barrel can be sold for $80 - x_1$ dollars. If Oilco extracts $x_2$ million barrels during year 2, each barrel can be sold for $85 - x_2$ dollars. The cost of extracting $x_1$ million barrels during year 1 is $2x_1^2$ million dollars, and the cost of extracting $x_2$ million barrels during year 2 is $3x_2^2$ million dollars. A total of 20 million barrels of oil are available and at most \$250 million can be spent on extraction.

Formulate a nonlinear model to determine how Oilco can maximize its profit (revenue less costs) for the next 2 years.

Problem 4

Mother Smith's Pies produces frozen fruit pies for sale to local restaurants. Demand for one of its best sellers, apple, averages 150 pies per day. The company's production facility is capable of producing 50 pies per hour and operates 8 hours a day, 365 days a year. The pies cost \$2.25 to produce and Mother Smith's estimates that its annual holding cost rate is 20%. Whenever a new pie type is produced the machinery must be thoroughly cleansed so the cost of a production setup is estimated to be \$90.

a. (5 pts) Explain how you would determine $Q^*$. Your explanation should include the formula(s) you would use to determine $Q^*$. Clearly indicate the values for each of the variables used in the formula(s). **NOTE:** You do not actually need to compute $Q^*$.

b. (10 pts) The time between starts of any two apple pie production runs can be divided into two distinct time periods. Explain what these two pieces represent and how you would calculate the length of each.

c. (10 pts) When does the inventory reach its highest level? The highest inventory level is what fraction of $Q^*$?

Problem 5

A couple has enough wire to construct 100 ft of fencing. They wish to use it to form 2 sides of a triangular garden, one side of which is along a building of side length 18 ft as shown below.

The area of a triangle with sides of length $a$, $b$, and $c$ is given by the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where

$$s = \frac{a+b+c}{2}.$$

Use calculus to determine the dimensions of the triangle that encloses the largest possible area.