Deterministic Math Models (550.251)
Homework 8 (Due Thursday, May 01, 2008)

General Directions: You must show all work and document any assumptions to receive full credit. Solve by hand unless otherwise stated. Please include a mathematical formulation of your model. When formulating models, make sure you define your variables and label all constraints.

1. *CSI-Baltimore.* A murder victim is discovered at midnight and the temperature of the body is recorded at 31 °C. One hour later, the temperature of the body is 29 °C. Assume that the surrounding air temperature remains constant at 21 °C. Use Newton's Law of Heating and Cooling to calculate the victim's time of death. *Note:* The "normal" temperature of a living human being is approximately 37 °C.

2. Suppose that a radioactive substance decays according to the model

$$\frac{dN}{dt} = -\lambda N \quad N(0) = N_0$$

where $N(t)$ is the amount of substance remaining at time t, and λ is the decay constant.

   (a) Show that the half-life of the radioactive substance is given by the equation

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

   (b) The half-life of ^{238}U is 4.47 $\times 10^7$ years. Determine its decay constant.

   (c) Suppose that 1000 mg ^{238}U are present initially. How long will it take for this sample to decay to 100 mg?

3. Suppose that the number $x(t)$ (with t in months) of alligators in a swamp satisfies the differential equation

$$\frac{dx}{dt} = 0.001x^2 - 0.01x$$

   (a) Solve this differential equation.

   (b) If $x(0) = 25$, determine what happens to the alligator population in the long run.

   (c) If $x(0) = 150$, determine what happens to the alligator population in the long run.

4. One the day of his birth, Jason's grandmother pledges to make available $50,000 on his eighteenth birthday for his college education. She negotiates an account paying 6.25% annual interest, compounded continuously, with an initial deposit of $1000, and agrees to deposit a fixed amount each year.

   (a) Formulate a differential equation that models this problem.

(b) What annual deposit should be made so she can reach her goal?

5. Infusion of glucose into the bloodstream is modeled by the first order ordinary differential equation

$$\frac{dG}{dt} = c - aG$$

where $G(t)$ is the amount of glucose in the bloodstream at time t, c is the cost rate of infusion, and a is a positive constant governing the removal rate from the bloodstream.

(a) Solve for $G(t)$ using the fact that the equation is separable.

(b) Solve for $G(t)$ using the fact that the equation is linear.

(c) Compute

$$\lim_{t\to\infty} G(t)$$

6. **(10pts Extra Credit)** Solve the following system of first order linear ODEs by first solving for $x(t)$ and then solving for $y(t)$:

$$\begin{aligned} x' &= -0.05x & x(0) = 50 \\ y' &= 0.5x - 0.025y & y(0) = 50 \end{aligned}$$